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AUGUST, 1921

Proceedings of the American Physical Society 85

Minutes of the One Hundred and Ninth Meeting. Crystal Structure of Titanium, Zirconium, Cerium, Thorium and Osmium, *Albert W. Hull;* The Width of X-Ray Spectral Lines, *L. Gilchrist;* Characteristic Low Voltage X-Radiation from Arcs in Metallic Vapors, *F. L. Mohler and Paul D. Foote;* Secondary High Frequency Radiation, *A. H. Compton;* Polarization of Secondary X-Rays, *A. H. Compton and C. F. Hagenow;* A Remeasurement of the Radiation Constant, by Means of X-Rays, *William Duane, H. H. Palmer and Chi-Sun Yeh;* Soft X-Rays of Characteristic Type, *E. H. Kurth;* The Change of the Mobility of the Positive Ions in Air with Age, *Henry A. Erikson;* The Absolute Sizes of Certain Monovalent Ions, *Wheeler P. Davey;* Forces Within a Static Atom, *Irving Langmuir;* Effect of Roughening of Surface on the Dispersion of Cannon Ball in Falling Through Air, *F. C. Brown;* On The Determination of Instantaneous Pressures in Guns by Means of a Piezo-Electric Gage and a Ballistic Galvanometer, *J. C. Karcher;* A New General Law of

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THE PHYSICAL REVIEW.

PROCEEDINGS OF THE AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE 109TH MEETING, WASHINGTON, APRIL 22 AND 23, 1921

THE 109th regular meeting of the American Physical Society was held at the Bureau of Standards in Washington, D. C., on Friday and Saturday, April 22 and 23, 1921. There were four sessions for the reading of papers. President Lyman presided and there were about 250 members and friends in attendance.

At the meeting of the Council held on April 22, 1921, fourteen persons were *elected to membership*, as follows: E. C. Bain, Ben C. Bellamy, James M. Douglas, Arthur R. Bauder, George Dobson, H. E. Fox, Charles F. Hagenow, L. E. Hildebrand, E. H. Kurth, J. W. Miles, Charles L. Swisher, J. R. Jenness, Victor C. Stechschulte, John R. Weeks, Jr.

It was voted that the Society coöperate with the International organization which has charge of the publication of Annual Tables of Constants.

The program contained 64 papers, twelve of which were read by title only. The program is given below:

Crystal Structure of Titanium, Zirconium, Cerium, Thorium and Osmium. ALBERT W. HALL.

The Width of X-Ray Spectral Lines. L. GILCHRIST.

Characteristic Low Voltage X-Radiation from Arcs in Metallic Vapors. F. L. MOHLER and PAUL D. FOOTE.

Secondary High Frequency Radiation. A. H. COMPTON.

Polarization of Secondary X-Rays. A. H. COMPTON and C. F. HAGENOW.

A Remeasurement of the Radiation Constant, by Means of X-Rays. WILLIAM DUANE, H. H. PALMER and CHI-SUN YEH.

Soft X-Rays of Characteristic Type. E. H. KURTH.

The Decrease of the Mobility of the Positive Ions in Air with Age. HENRY A. ERIKSON.

The Absolute Sizes of Certain Monovalent Ions. WHEELER P. DAVEY.

Forces Within a Static Atom. IRVING LANGMUIR.

Effect of Roughening of Surface on the Dispersion of Cannon Ball in Falling Through Air. F. C. BROWN.

On The Determination of Instantaneous Pressures in Guns by Means of a Piezo-Electric Gage and a Ballistic Galvanometer. J. C. KARCHER.

A New General Law of Deformation. P. G. NUTTING. (Read by title.)

Contractions and Expansions of Amalgams with Time. ARTHUR W. GRAY.

Measurement of Mercury Vapor Pressure from 0° to 35° by the Knudsen Pressure Gauge. C. F. HILL. (Read by title.)

Thermal Conductivity of White Marble and "Neat" Cement. ROY A. NELSON. (Read by title.)

The Effect of Pressure on the Thermal Conductivity of Metals. P. W. BRIDGMAN.

The New Fixed Point on the Thermometric Scale. FREDERICK BATES and F. P. PHELPS.

A Disappearing Filament Optical Pyrometer Free from Diffraction Effects at the Filament. C. O. FAIRCHILD.

The Organ Pipe as a Coupled System. ARTHUR TABER JONES.

On the Measurement of the Acoustic Output and Efficiency of Fog-alarm Apparatus. LOUIS V. KING.

A Simple Apparatus for Testing Pitch Control. PRESTON EDWARDS.

Some New Consequences of the Theory of Ultimate Rational Units. GILBERT N. LEWIS.

Present Status of the Electric and Magnetic Units. J. H. DELLINGER.

Discussion of a Kinetic Theory of Gravitation II., and Some New Experiments in Gravitation. CHARLES F. BRUSH.

Defects in Symmetric Optical Systems. IRWIN ROMAN. (Read by title.)

The High-Frequency Resistance of Inductance Coils. G. BREIT. (Read by title.)

A Method of Measuring Coil Capacities and Standardizing Wavemeters. G. BREIT. (Read by title.)

Tables for the Calculation of the Inductance of Circular Coils of Rectangular Cross Section. FREDERICK W. GROVER. (Read by title.)

On Some New Formulæ for the Direct Numerical Calculation of the Coefficient of Mutual Induction of Coaxial Circles. LOUIS V. KING.

Note on the Earth's Magnetic Field. W. F. G. SWANN.

The Electrical Resistance of the Human Body. N. FORMAN, J. S. MARTIN and F. WENNER.

New Methods for Maintaining Constant Frequency in High-frequency Circuits. W. G. CADY.

Magnetic Rotatory Dispersion in Transparent Liquids. R. A. CASTLEMAN, JR., and E. O. HULBURT.

A Direct Reading Spectrophotometer for Measuring the Transmissivity of Liquids. IRWIN G. PRIEST.

The Excitation of Enhanced Spectra in Low Voltage Arcs. PAUL D. FOOTE, W. F. MEGGERS and F. L. MOHLER.

On the Photoelectric Effect of Alkali Vapors. JAKOB KUNZ.

Photo-Electric Potentials from the Retina. E. L. CHAFFEE and W. T. BOVIE.

The Variation of Metallic Conductivity with Electrostatic Charge. HENRY A. PERKINS.

The Distributed Capacity of Inductance Coils. G. BREIT.

Some Effects of the Distributed Capacity of Coils to Ground. G. BREIT. (Read by title.)

Total Emissive Powers and Resistivities of Tungsten at Incandescence. A. G. WORTHING and W. E. FORSYTHE.

Color Temperature of High Efficiency Lamps. W. E. FORSYTHE.

The Detecting Efficiency of the Resistance-Captivity Coupled Electron Tube Amplifier. E. O. HULBURT.

Radio Signal Fading Phenomena. J. H. DELLINGER and L. E. WHITTEMORE.

Some Physical Problems of Aircraft Radio. L. E. WHITTEMORE.

The Design of a Standard Wavemeter. R. T. COX.

The High-Frequency Impedance of Radio Telephone Receivers. C. T. ZAHN.

A Record of Radio Time Signals. E. A. ECKHARDT and J. C. KARCHER.

An International Series of Radio Audibility and Directional Measurements. CARL KINSLEY.

Detection Coefficient of Thermionic Valves. E. L. CHAFFEE.

The Radio Research Field. J. H. DELLINGER and L. E. WHITTEMORE.

On Emergent β Rays from Materials of Different Atomic Weight and Different Thickness Produced by γ Rays; Studies by the Statistical Method. ALOIS F. KOVARIK. (Read by title.)

High Frequency Limits of X-Ray Spectra at Different Angles with the Cathode Stream. D. L. WEBSTER.

A Generalization of the Electrodynamic Equations with Applications to the Structure of the Electron and Radiationless Orbits. LEIGH PAGE.

Resistance and Inductance of a Three-Phase Cable. HARVEY L. CURTIS.

Thickness of Stratified Soap Films. P. V. WELLS. (Read by title.)

Temperature of an Electrically Heated Filament. CHESTER SNOW.

Standard Wave Lengths and the Constant Frequency Differences in the Spectra of Higher Gases. W. F. MEGGERS.

Note on the Diurnal Variation of the Atmospheric Electric Potential Gradient. S. J. MAUCHLY.

Peculiar Ice Formations. H. G. DORSEY.

On the Exploded Wire Spectrum of Calcium. R. A. SAWYER and A. E. BECKER.

On the Radiation of Energy from Coils. A. G. WEBSTER. (Read by title.)

On the Vibration of Gun Barrels. A. G. WEBSTER. (Read by title.)

GEORGE B. PEGRAM,
Secretary pro tem.

CRYSTAL STRUCTURE OF TITANIUM, ZIRCONIUM, CERIUM, THORIUM AND OSMIUM.

BY ALBERT W. HULL.

X-RAY powder diffraction patterns of these metals were taken in the manner previously described,[1] using a molybdenum water-cooled X-ray tube, with zirconium filter. The positions of the lines on the patterns were measured on a slide rule, and the corresponding planar spacings calculated by the Bragg formula.

Since there is no crystallographic data regarding these substances, it was necessary to consider all possible crystal systems and axial ratios. For the hexagonal and tetragonal systems (including cubic) plots were available[2] giving theoretical planar spacings as a function of axial ratio. Fortunately these plots were found to be sufficient, four of the metals being hexagonal, and the fifth cubic. The entire process of discovering the crystal system, axial ratio, type of lattice, and dimensions of the lattice was thus accomplished in a few minutes.

Titanium was found to crystallize in the hexagonal system, holohedral class, with axial ratio $1.59 \pm .01$. The arrangement of atoms is a slightly distorted form of "hexagonal close packing" of solid spheres, being shortened in the direction of the principal axis in the ratio 1.633 : 1.59. It is a close packed arrangement of oblate spheroids whose axes of revolution are 2 per cent. shorter than the lateral axes.

The side of the unit triangular prism is 2.97 Å, and its height 4.72 Å. The lattice is made up of two sets of these triangular prisms, the atoms of one set being in the center of the prisms of the other set.

[1] PHYS. REV., 10, 661, 1917.
[2] PHYS. REV., 17, 549, 1921.

Zirconium has a crystal structure identical with Titanium, with axial ratio also 1.59. The side of its unit triangle is 3.23 Å and its height 5.14 Å.

Cerium shows the same structure as Ti and Zr, with axial ratio 1.62. The accuracy of the data is not sufficient in this case to distinguish with certainty between the experimental value 1.62 and true hexagonal close packing of spheres, 1.633. The side of the elementary triangle is 3.65 Å, and height 5.96 Å (taking $c = 1.633$).

There is present also a face centered cubic form, with side of cube 5.12 Å giving the same density as the hexagonal form. Cerium thus appears to be composed of a mixture of the hexagonal and cubic forms of close-packed arrangements of atoms, like cobalt. The effect of annealing has not been tried. It is possible, however, that the apparent dimorphism is due to impurity, as the sample, though supposed to be very pure, has not been analyzed and its pattern was almost identical with that of "misch metal," which contains about 8 per cent. iron. Further investigations are in progress.

Thorium is a face-centered cubic lattice with side of cube 5.04 Å.

The atoms of these four metals are very similar in structure, each being composed of a stable symmetrical shell (inert gas) surrounded by four electrons. In the case of the lighter elements of similar structure, C and Si, the four external electrons give perfect tetrahedral symmetry to the atom, as evidenced both by chemical properties and crystal structure. With the heavier metals here investigated the effect of the four outer electrons is much less marked. In the case of Ti and Zr, the crystal structures indicate that the four outer electrons merely increase the peripheral dimensions of the spherical shell, making it a slightly oblate spheroid; while in the case of Ce and Th the spherical symmetry is not affected at all—the atoms pack like perfect spheres.

Osmium is likewise found to crystallize in a hexagonal close packed lattice, with axial ratio 1.59. The side of the unit triangle is 2.714 Å, and its height 4.32 Å.

The analysis of osmium completes the eighth group metals. The results may be briefly summarized:—The second and third elements of each row, viz., Co, Ni, Rh, Pd, Ir, Pt, are *face centered cubic arrangements*. The first members, Fe, Ru and Os, are different from the rest. Fe is *body centered cubic;* Ru and Os are *hexagonal close packed arrangements of oblate spheroids*, the axial ratio of each being 1.59.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
April 5, 1921.

THE WIDTH OF X-RAY SPECTRAL LINES.

BY L. GILCHRIST.

SINCE certain instrumental factors which affect the width of spectral lines of ordinary light are not effective in the case of spectral lines, it would appear to be reasonably justifiable to investigate the conditions that might exist in

the source of the x-rays, *i.e.*, in the target of the x-ray tube which would give the line width.

A consideration of the Doppler effect which was presented some time ago led to a value for the velocity of the source which was much too high, or conversely if the velocity of the source in the target is lower than that of an atom of gas, as given by the kinetic theory the width produced thereby would be inappreciable and would probably be masked by the widening produced from other causes.

The assumptions are now made that

(*a*) The x-rays are produced only by the impinging electrons which penetrate close to the nucleus of the atom to approximately 10^{-12} cm. from the center.

(*b*) The wave-length produced is given by

$$\text{P.D.} \times e = h\gamma = \frac{hc}{\lambda}$$

or

$$\tfrac{1}{2}mv^2e = \frac{hc}{\lambda},$$

where m = mass of electrons,
v = velocity of electrons,
c = velocity of x-rays.

(*c*) The line is widened because of the motion of the atom so that

$$(\tfrac{1}{2}M\xi^2 + \tfrac{1}{2}mv^2)e = \frac{hc}{\lambda_1}$$

where ξ is the component of the velocity of the atom along the direction of the impinging electron.

I. To obtain maximum change in wave-length and the resolving power necessary to show it, use the formula—

$$\text{P.D.} \times e = h\gamma = \frac{hc}{\lambda} \text{ and P.D.} \times e = \tfrac{1}{2}mv^2,$$

where m = mass of the impinging electron,
e = charge of the electron,
v = velocity of the impinging electron.

Then

$$\tfrac{1}{2}mv^2e = \frac{hc}{\lambda}.$$

Then the changed wave-length is given by—

$$\tfrac{1}{2}mv^2e + \tfrac{1}{2}MV^2e = \frac{hc}{\lambda_1},$$

where V = the velocity of the atom in the target,
M = the mass of the atom.

$$\therefore \quad \frac{\frac{1}{2}MV^2e}{\frac{1}{2}mv^2e} = h\frac{\left(\frac{1}{\lambda_1}-\frac{1}{\lambda}\right)}{h\frac{1}{\lambda}} = \frac{\frac{\lambda-\lambda_1}{\lambda\lambda_1}}{\frac{1}{\lambda}} = \frac{\lambda-\lambda_1}{\lambda_1} = \frac{d\lambda}{\lambda_1};$$

or resolving power necessary to show this when a tungsten target is used is

$$\frac{\lambda_1}{d\lambda} = \frac{mv^2}{MV^2} = \frac{(10^{10})^2}{4 \times 184 \times 1800 \times 10^{10}},$$

if $V = 10^5$ cm. per sec. $M = 184 \times 1800 \times$ mass of the electron.

$$\therefore \frac{\lambda_1}{d\lambda} = \frac{10^8}{4 \times 184 \times 18} = \frac{10^4}{1.3248} = 7.5 \times 10^3,$$

i.e., the resolving power.

(*a*) $R = 7.5 \times 10^3$ which is necessary to show this width. If the velocity of the atoms is 10^4 cm. per sec. the resolving power necessary to show the maximum width is

(*b*) $R = 7.5 \times 10^5$ cm. per sec., or for (*a*) a change in λ of

$$d\lambda = \frac{1}{7.5 \times 10^3}$$

of λ_1 would have to be measurable or for (*b*) a change in λ of

$$d\lambda = \frac{1}{7.5 \times 10^5}$$

of λ_1 would have to be measurable. Even the latter would appear to be detectable and even measurable, unless the slits through which the x-ray beam passes are too wide. Even then it would appear that sections of a wedge-shaped slit could be used to eliminate the effect of the slit in much the same way as Merton at Oxford uses a wedge for spectral lines of ordinary light.

II. To obtain the effect of a change of temperature in the target from 300° absolute to 2100° absolute

$$\frac{\lambda_1}{d\lambda} = \frac{mv^2}{M\frac{2100}{300}V^2} = \frac{mv^2}{7MV^2}$$

i.e.,

$$d\lambda = \frac{7}{7.5 \times 10^3} \text{ of } \lambda_1$$

or 7 × as large as in 1 (*a*) or 1 (*b*).

III. To obtain the distribution of intensity in the x-ray spectral line which will be a representation of the distribution of intensity in the source of the x-ray, the expression for the intensity is

$$I = 2\int_{\lambda_1}^{\lambda_2} \psi(\lambda)\left(1 + \cos 4\pi \frac{D}{\lambda}\right) d\lambda$$

where $2D$ is the path difference between the interfering beams.

Now let Λ = natural wave-length N = natural frequency.

λ = actual wave-length n = actual frequency.

Now

$$\frac{\lambda}{\Lambda} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}M\xi^2}{\frac{1}{2}mv^2} = 1 - \frac{M}{m}\frac{\xi^2}{V^2},$$

where ξ is the component velocity of the atom in the direction of the moving electron, or

$$\frac{\Lambda}{\lambda} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 - \frac{1}{2}M\xi^2} = \left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right),$$

or

$$\frac{\frac{1}{\lambda}}{\frac{1}{\Lambda}} = \left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right),$$

or

$$\frac{1}{\lambda} = \frac{1}{\Lambda}\left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right);$$

$$\therefore \frac{I}{2} = \int_{-\infty}^{+\infty} e - \beta\xi^2 \left\{1 + \cos 4\pi \frac{D}{\Lambda}\left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right)\right\} d\xi$$

$$= \int_{-\infty}^{+\infty} e - \beta\xi^2 \left\{1 + \cos 4\pi \frac{D}{\Lambda} \cos\left(4\pi \frac{D}{\Lambda}\frac{M}{m}\frac{\xi^2}{V^2}\right)\right\} d\xi.$$

Now

$$\int_{-\infty}^{+\infty} - \beta\xi^2 d\xi = \sqrt{\frac{\pi}{\beta}}.$$

Now put in

$$V = \frac{2}{\sqrt{\pi\beta}} \text{ or } \sqrt{\beta} = \frac{2}{\sqrt{\pi}V} \text{ or } \beta = \frac{4}{\pi V^2},$$

where

V = the root mean square velocity of the atom.

Then

$$I = \pi V + \cos 4\pi \frac{D}{\Lambda}\sqrt{\frac{\frac{\pi}{2}\frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}$$

$$+ \sin 4\pi \frac{D}{\Lambda}\sqrt{\frac{\frac{\pi}{2} - \frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}. \qquad (5)$$

From this V cannot be evaluated directly since the amplitude of the x-ray pulse at its source has been arbitrarily put equal to unity. But when $2D = o\,I$ is a max, then

$$I \max = \pi V + \sqrt{\frac{\frac{\pi}{2}\frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{\Delta}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)}}{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{\Delta}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}$$

$$= \frac{3}{2}\pi V.$$

And if $2D_1$ = path difference where

$$I = \frac{1}{10} I \max.$$

as shown by the spectral line. Then

$$\frac{3}{20}\pi V = \pi V + \cos 4\pi \frac{\Delta_1}{\Lambda}\left\{\sqrt{\frac{\frac{\pi}{2}\frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{\Delta_1}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{\Delta_1}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}\right.$$

$$+ \sin 4\pi \frac{\Delta_1}{\Lambda}\left\{\sqrt{\frac{\frac{\pi}{2} - \frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{\Delta_1}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi \frac{\Delta_1}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}\right. . \qquad (6)$$

From equation (6) V may be determined since the values of all the other terms are known.

This equation may be somewhat simplified by certain modifications and approximations but this may be left in abeyance until it is known whether the actual width of the source may be determined from the spectral line and whether the temperature effect on the source may be detected by a corresponding change in the width of the spectral line.

Observations on the L series of spectral lines seem to give a little better promise of measurable results than observations on the K series of lines, for if all the energy of the impinging electrons goes into the x-ray pulse produced, then, according to Whiddington[1] the velocity of the electrons is given by

$$v = (N - 15)10^8 \text{ cm. per sec.},$$

where N is the atomic number of the material in the target. For tungsten $N = 74$. Then the required resolving power in I (a) becomes $R = 2.5 \times 10^3$, and in I (b) becomes $R = 2.5 \times 10^5$.

D. L. Webster[2] shows that the relationship $P.D. \times e = h\nu$ does not hold as a relationship connecting the characteristic radiations or their components.

[1] Phil. Mag., 39, June, 1920, pp. 694–696.

[2] PHYS. REV., VII., No. 6, July, 1916, pp. 599–613.

It is not assumed in these calculations that this relationship holds. But it is assumed that the small width which any characteristic line has is of the nature of a general radiation to which the equation will apply.

Further it would appear that the velocities of the atoms in the target as calculated from equation (6) are still too high even though they are much lower than those which were obtained on the assumption that the width of the line is due to the Doppler effect.

The following modifications have therefore been made

(*a*) The characteristic x-ray spectra lines are produced by a disturbance of the electrons in the inner orbits of the atom, *i.e.*, the only orbits which exist in the case of hydrogen, helium, lithium.

(*b*) The ordinary ultra-violet, visible, and infra-red spectral series of hydrogen are its characteristic x-ray spectral groups.

(*c*) It is assumed that the critical velocity of the electrons necessary to produce the characteristic lines of Tungsten is of the order of 10^{10} cm. per second and the corresponding velocity necessary to produce the characteristic lines of hydrogen is of the order of 10^{7} cm. per second. This is supported by the speed of the electrons emitted by a metal when these radiations fall upon it.

(*d*) The further assumption is made that this critical velocity of the electrons, viz., 10^{7} cm. per second, which is required to produce hydrogen lines, is the same for all substances, but in the case of those atoms with a large number of electronic orbits about the nucleus it is necessary that the impinging electrons should have a higher velocity than this in order that they may penetrate to the inner orbits, and in the process of penetration their velocity is decreased in a manner similar to that of β particles shot into a cloud forming atmosphere. The critical velocity of the electrons for the production of x-rays is $\therefore = 10^{7}$ cm. per sec. and is independent of the nature of the target.

Some of the spectral lines given by William Duane,[1] have been examined and their widths and the angular distance between the successive orders measured and by means of equation (6) the velocities of the atoms in the targets have been calculated.

The results show that the velocities are of the order—12 to 14 meters per second.

UNIVERSITY OF TORONTO,
April 2, 1921.

CHARACTERISTIC LOW VOLTAGE X-RADIATION FROM ARCS IN METALLIC VAPORS.

BY F. L. MOHLER AND PAUL D. FOOTE.

A SEARCH for critical potentials above the resonance potentials and first ionization potentials has been made by the following methods. The metals

[1] PHYS. REV., XVI., No. 6, Dec., 1920.

were boiled in tubes containing four electrodes, a hot wire cathode surrounded by two grids and a cylindrical plate. A variable accelerating potential V between the cathode and inner grid maintained the arc. With the outer grid negative and the plate positive all electrons and ions are shielded from the plate and the photo-electric effect of the arc radiation on the plate can be measured as a function of V. With the plate negative the ionization can be measured. The plate current divided by the cathode current was plotted against the accelerating potential. It happens that the radiation voltage curves are then straight lines between critical potentials, so that the critical radiating potentials are shown very sensitively as points where the slope changes. Ionization points above the first are more difficult to locate. Initial potentials can be computed from the observed first ionization point. The following results have been obtained.

Potassium, a sharp break in the radiation curve at 23 ± 1 volts. With increased vapor pressure another break at 20 ± 1 volts. Ionization curves under some conditions show a break at 23 volts.

Sodium, a break in the radiation curve at 35 ± 1 volts much less pronounced than in potassium and not evident at low vapor pressure. A break was also evident in the ionization curves at 35 volts under some conditions.

Magnesium, a break in the radiation curve at 46 ± 1 volts. Above this the curve was a straight line. Below thirty volts there was marked curvature but the critical points were not measurable under the conditions of the experiments. The break at 46 volts was similar in intensity to that obtained in sodium.

The x-ray wave-length L_{α_1}, for magnesium can be computed by the law of Duane in the form $L_{\alpha_1} = K_\alpha - K_\alpha$. It gives L_{α_1} at $\lambda = 263$ Å or a potential of 46.9 volts. The observed value was 46. For sodium L_{α_1} can not be directly computed as K_α has not been observed. Extrapolation by the Moseley relation of values of L_{α_1} obtained as above gives about 30 volts. The observed value 35 ± 1 may point to a marked deviation from Moseley's law at the lower end of the L series. However the observed point may be the potential required to doubly ionize the atom which would be higher than the limit of the L series by approximately the ionization potential, 5 volts. The experimental value agrees exactly with this hypothesis.

Theories of atomic structure indicate that the critical potentials observed in potassium should be related to the M series. It is impossible to compute this frequency accurately from x-ray data. All we can say is that the order of magnitude is correct. The principal point at 23 volts corresponds to $\lambda = 535$ Å.

BUREAU OF STANDARDS, WASHINGTON, D. C.,
April 5, 1921.

SECONDARY HIGH FREQUENCY RADIATION.

BY ARTHUR H. COMPTON.

THE experiments performed by Florance and Ishino on the scattering of gamma rays showed a considerable difference in hardness between the primary and the secondary radiation, an effect difficult to explain if the secondary radiation was truly scattered. An experimental method of distinguishing between truly scattered and fluorescent radiation was therefore devised. This consisted of placing an absorbing screen alternately in the path of the primary and of the secondary beam. Any truly scattered radiation should remain of the same intensity for both positions of the absorbing screen, while the fluorescent rays, being in general softer than the primary rays exciting them, would be more strongly absorbed when the screen is in the path of the secondary radiation. The ratio of the truly scattered to the total secondary rays is the limit of the ratio of the intensity observed with the screen in the path of the secondary to that when the screen is in the path of the primary beam as the thickness of the screen is increased.

The experiments showed that with gamma rays the amount of truly scattered radiation at angles greater than 90° as determined by this criterion is a negligible part of the total secondary rays. At 45° about 3 per cent. of the radiation seems to be of the scattered type.

Similar experiments were performed with x-rays of wave-lengths between 0.12 and 0.50 Å.U. using a modified Bragg spectrometer with a recording device to register the intensity of the secondary radiation for different angles of the ionization chamber. These rays showed an effect identical in character with that observed with gamma rays, but not so prominent. With $\lambda = 0.12$ Å.U. the scattered rays form about 15 per cent. at 90° with the primary beam and 70 per cent. of the total radiation at 30°. For $\lambda = 0.50$ the scattering is relatively much more important, but the fluorescent radiation is still a large part of the total radiation. These results, both in the case of the gamma rays and of the x-rays, seem to be independent of the material used as radiator except in those cases where a perceptible amount of characteristic fluorescent radiation enters the ionization chamber.

The experiments therefore indicate the existence of a type of fluorescent high frequency radiation whose wave-length is independent of the particular substance used as radiator, depending only on the frequency of the exciting primary rays.

The measurements made by Barkla, Crowther, Owen and others represented the total secondary radiation, only a part of which, according to the present measurements, represents truly scattered rays. A direct measurement of the absolute value of the scattering shows that at small angles the scattering of x-rays approaches that predicted according to Thomson's theory if the number of effective electrons per atom is equal to the atomic number. At large angles however, the scattering is much less than predicted by this theory, especially

for the shorter wave-lengths, being less than 1/1,000th of the theoretical value in the case of gamma rays. The results are in much better accord with the hypothesis that the electron is of sufficient size (about 3×10^{-10} cm.) to give rise to interference effects between the rays scattered from its different parts.

WASHINGTON UNIVERSITY,
SAINT LOUIS, April 21, 1921.

THE POLARIZATION OF SECONDARY X-RAYS.

BY A. H. COMPTON AND C. F. HAGENOW.

IN the previous paper by one of us, evidence is presented for the existence of a type of fluorescent radiation whose wave-length is independent of the material of the radiator, being a function only of the wave-length of the primary radiation. The explanation suggests itself that this general fluorescent radiation may be the x-rays excited by the impact of the secondary cathode rays liberated in the radiator by the action of the primary radiation. While such an explanation accounts for many of the characteristics of this radiation, quantitative measurements showed that at least in some cases as much as 50 per cent. of the energy of the primary rays appeared as general fluorescent radiation. Since the efficiency of production of x-rays by cathode rays is less than 1 per cent., it does not seem that this can be the real origin of the secondary rays. It occurred to us that some information on this point might be obtained from a careful measurement of the polarization of the fluorescent radiation.

Barkla, in his classic measurement of the polarization of secondary x-rays, found that at 90° the rays were approximately 80 per cent. polarized. The remaining 20 per cent. might be accounted for in part by experimental error, and the remainder Barkla ascribed as due possibly to a real lack of polarization due to forces acting on the scattering electrons. We employed much more sensitive methods of measuring the tertiary radiation than those used by Barkla and his followers. The first radiator was placed about 12.5 cm. directly above the second radiator, which was in turn placed on the crystal table of a Bragg spectrometer. The degree of polarization could then be measured by noting the scattering from the second radiator parallel and at right angles with the primary beam. The geometrical conditions were such that at the 90° position about 4 or 5 per cent. of the radiation should appear to be unpolarized (the apertures being so large that the scattering angles were not exact right angles).

For thick blocks of scattering material our results confirmed those of Barkla, It was found, however, that when thin sheets of the radiators were employed. the polarization became much more complete. As one might have anticipated, multiple scattering in the case of the thick radiators was responsbile for a considerable lack of polarization. Extrapolating our results to zero thickness we find the ratio of the intensity of the tertiary rays at 90° to that at 0° to be for a paper radiator, 0.05, for aluminium, 0.04 and for sulphur, 0.05. When

these results are corrected for the imperfection of the geometrical conditions, it means that the polarization is complete within experimental error.

Since under the conditions of our experiment approximately 70 per cent. of the total secondary radiation at 90° is of the general fluorescent type, this result means that the fluorescent as well as the scattered rays are completely polarized. Barkla's experiments show that the general radiation from an x-ray tube is only partly polarized, which makes it difficult to support the view that the general fluorescent radiation is excited in exactly the same manner as the primary x-rays. The hypothesis that this radiation is emitted at the instant of liberation of the secondary cathode rays from the atoms, however, seems to offer a possible explanation of both the degree of polarization and the efficiency of production of these fluorescent rays.

WASHINGTON UNIVERSITY,
SAINT LOUIS, April 21, 1921.

A REMEASUREMENT OF THE RADIATION CONSTANT, h, BY MEANS OF X-RAYS.

BY WILLIAM DUANE, H. H. PALMER AND CHI-SUN YEH.

SINCE the discovery of the fact[1] that the quantum law, $Ve = h\nu$, applies to the short wave-length limit of the continuous x-ray spectrum, a number of scientists have used the law in measurements of the value of h.[2] The most accurate measurement of h made in our laboratory by this method[3] gave the value $h = 6.555 \times 10^{-27}$. In this research we measured the voltage, V, by means of an electrostatic voltmeter, which we calibrated by passing a known current through a metallic resistance of several hundred thousand ohms.

Several years ago our laboratory purchased a series of resistance coils aggregating 10,000,000 ohms for the purpose of comparing high voltages directly with the E.M.F. of a standard cell by means of the simple potentiometer principle. The object of the research reported in this paper has been to measure the value of h using this high resistance potentiometer to measure the voltage, V. We took extreme care in insulating the various electrical circuits and instruments, and in determining the *ratios* of the several resistances. Two Weston cadmium cells served as standards of E.M.F., for each of which the Bureau of Standards had furnished a certificate, the error mentioned in the certificates being .01 per cent. Considering the fact that the ratios of resistances can be determined with great precision, and that we had two standard cells at our disposal, it seems probable that our error in measuring the voltage, V, did not exceed .01 per cent. This means that the error in V is negligible as compared with those in the other quantities entering into the above quantum equation.

In measuring the wave-lengths (and frequencies, ν) we used a somewhat better x-ray spectrometer than that employed before, and a different calcite

[1] Duane and Hunt, PHYS. REV., Aug., 1915, p. 166.

[2] See "Data Relating to X-Ray Spectra," by William Duane, published by the National Research Council.

[3] Blake and Duane, PHYS. REV., Dec., 1917, p. 624.

crystal. The x-ray tube had a thin mica window attached to it through which the x-rays emerged. This diminished the absorption of the x-rays, and enabled us to use a narrower spectrometer slit. We thereby reduced the correction for the slit's width to about 45″ of arc.

The glancing angle of incidence, θ, could be determined in a good experiment to within about 7″. As θ averaged about 290′ this means a precision of one part in 2,500.

As the weighted mean value of $V \sin \theta$ in five complete experiments we obtained

$$V \sin \theta = 2040 \pm 1.$$

Our 1917 value of $V \sin \theta$ was 2040 ± 3 about.

Taking Millikan's value of $e = 4.774 \times 10^{-10}$ e.s.u. and the value $2d = (6.056 \pm .004) \times 10^{-8}$ cm.for the grating constant of calcite we get for the value of h

$$h = (6.556 \pm .009) \times 10^{-27}.$$

This does not differ from the value previously obtained by as much as the errors of experiment. It is a more accurate value, however, in that the error in the voltage measurement has been eliminated.

An interesting series of measurements of h by means of x-rays has been made recently by Wagner. His most recent value is 6.52×10^{-27}, which differs from ours by about half of one per cent.

Experiments with x-rays coming from the tube at an angle of 45° from the stream of cathode rays indicate that the value of h is the same for them as for x-rays at right angles to the stream.

HARVARD UNIVERSITY,
April 5, 1921.

SOFT X-RAYS OF CHARACTERISTIC TYPE.

BY E. H. KURTH.

THIS paper covers some further results secured in the investigation which I reported upon at the last meeting of this society.

Radiations from several other elements including oxygen have been investigated. The latter element was worked with in the compound copper oxide.

Radiation corresponding to an x-ray series of the N type has been definitely observed for copper and iron. This series will be further investigated for elements of higher atomic number.

In the following table the small corrections for the filament drop and for the

Element.	K Series.	L Series.	M Series.	N Series.
Carbon.....	42.6 Å	375 Å		
Oxygen.....	23.8 Å	248 Å		
Iron.....		16.3 Å	54.3 Å	247 Å
Copper.....		12.3 Å	41.6 Å	119 Å

initial velocity of the electrons have been ignored in computing the wave-lengths because of the uncertainty regarding the correction to be applied for the energy involved in the absorption of the electrons by the target.

PRINCETON, UNIVERSITY.

THE CHANGE OF MOBILITY OF THE POSITIVE ION IN AIR WITH AGE.

BY HENRY A. ERIKSON.

AT the December, 1920, meeting[1] of the American Physical Society, the writer reported results indicating that there is very little difference in the mobility of the positive and negative ions in air if the mobility is measured. immediately after the ions are formed but that after an interval of the order of a second the mobilities assume the normal difference giving a ratio of 1.4.

The following is a report of the results ot a further investigation of this effect.

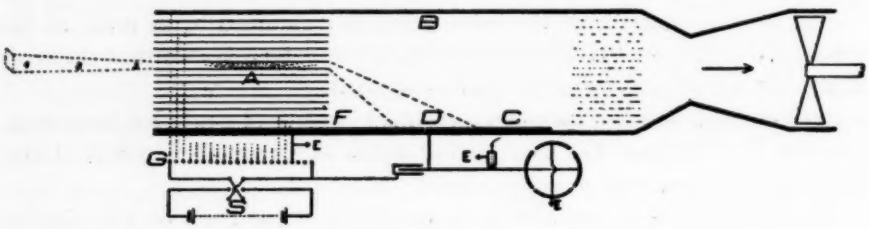

Fig. 1.

The apparatus used was essentially the same as in the earlier investigation Ions prduced at *A*, Fig. 1, by means of polonium are passed between two parallel plates *B* and *C* by means of a current of air. The field between *B* and *C* separates the ions. The number of ions arriving at the different down stream distances on the lower plate *C* is determined by means of a narrow strip *D* which is connected to a quadrant electrometer. The apparatus was arranged so that the distance *DF* could be changed without altering the field or the velocity of the air. In order to secure a steady flow of air a fan driven by a synchronous motor was used and a series of parallel plates were inserted above and below the plates between which the ions were produced. These

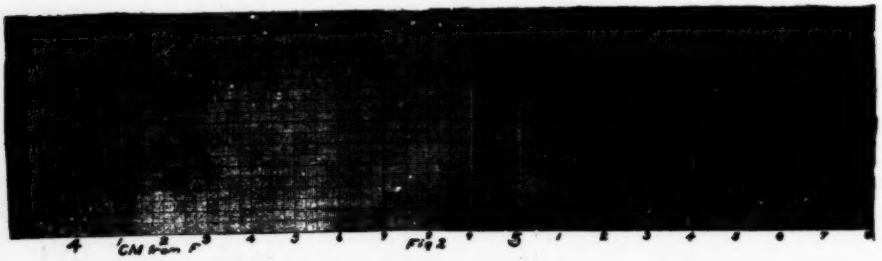

Fig. 2.

[1] PHYS. REV., XVII., p. 400.

plates were connected to points of increasing potential as shown at *G*. The uniformity of the field was secured by means of wires connected to the plates and passing parallel on the two sides of the stream of air. An investigation was first made to determine if there is any change in the mobility of the negative ion with age or if the change takes place in the positive only. An extension was inserted so that the ions could be produced at points indicated by *A*, 2, 3, 4, Fig. 1. An air velocity such as to give an average age of 0.2 seconds for point *A* up to one second for point 4 was used. It is seen from Fig. 2, that the shift takes place in the positive ion only. The irregular shift of the negative curve is undoubtedly due to turbulence and experimental error. In these curves the scale for the width of each curve was arbitrarily diminished.

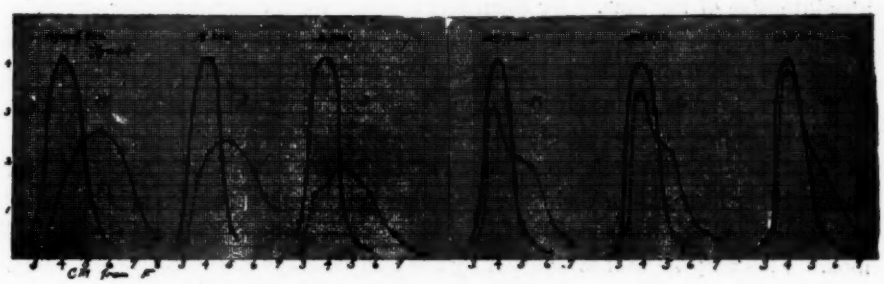

Fig. 3.

The next point investigated was to determine the nature of the shift in the case of the positive ion. The wind velocity was adjusted so as to give an average life of 0.8 seconds and the curves shown at *H*, Fig. 3, were obtained. The full curve represents the negative and the dotted curve the positive ion. Both curves were obtained simultaneously by reversing the field at *S*, Fig. 1. By using the maxima for these curves the normal mobility ratio of 1.4 is obtained.

By increasing the wind velocity so as to give an age of .4 seconds, the curves *I* were obtained. These show a shift of the positive towards the left, *i.e.*, the positive mobility has increased.

The curves *J* are for an age of 0.2 seconds and show a further shift of the positive to the left.

The curves *K* are for an age of 0.08 seconds. Here there was a further shift of the positive towards the left and an offset in the curve came into evidence. Curves *L* are for an age of .05 seconds. For an age of .03 seconds, curves *M*, the mobility of the positive ion has become essentially the same as that of the negative. A small offset is still present extending beyond the negative curve.

Owing to the loss of ions by recombination the currents measured diminished as the time increased. In the figure the ordinates have been made alike.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
April 16, 1921.

THE ABSOLUTE SIZES OF CERTAIN MONOVALENT IONS.

BY WHEELER P. DAVEY.

PROF. W. L. BRAGG has shown[1] from x-ray crystallographic data, that the difference in size between two ions is approximately constant and is independent of the state of combination of the ions as long as the valences are unaltered. The dimensions of the unit crystals alone, however, are not sufficient to determine the absolute size of the component ions, for there are always fewer equations than unknowns. Bragg attempted to use the dimensions of the crystals of elements as a means of elimination of one of the unknowns, and arrived at a remarkable series of numbers which he believed to represent approximately the absolute size of the ions. By assuming a structure for certain other crystals, he predicted from their densities what the dimensions of their unit crystals should be, and from this the size of the ions. The results seemed to be quite consistent among themselves, but did not agree with experimental data in this laboratory on the crystals themselves.[2]

Bragg's calculations are open to the objection that the atoms of metals are considerably larger than their corresponding ions, so that the size of the atoms cannot be substituted correctly in an equation dealing only with the size of ions. This may be well illustrated by the fact that the radius of the Ca atom, as shown by the crystal structure of the element, is 3.93 Å,[3] while the distance from $\overset{++}{Ca}$ to $\overset{++}{Ca}$ centers in CaO is only 3.42 Å.[4]

It seemed therefore desirable to recalculate the size of ions, using only assumptions dealing with ions, rather than atoms. The assumption chosen was, that $\overset{+}{K}$, $\overset{+}{Rb}$ and $\overset{+}{Cs}$ were equal in size to $\overset{-}{Cl}$, $\overset{-}{Br}$ and $\overset{-}{I}$, respectively. This assumption is in agreement with Langmuir's theoretical deductions on the volumes of ions of elements having nearly the same atomic numbers as the inert gases.[5]

From the standpoint of diffraction of x-rays, it may be justified as follows. Since KCl gives the diffraction pattern of a single cube of ions, the scattering powers of $\overset{+}{K}$ and $\overset{-}{Cl}$ must be approximately equal. Since, in addition to this, the number of electrons in the ions is the same, the ions must be of nearly the same size, *i.e.*, must have nearly the same *density* of electrons. If the nuclear charges had been identical in amount, then the two ions would have been of exactly the same size. This condition is approximated more nearly in RbBr and CsI. Bragg's values for $\overset{+}{K}$ and $\overset{-}{Cl}$ and for $\overset{+}{Rb}$ and $\overset{-}{Br}$ differ by almost 100 per cent. His value for $\overset{+}{Cs}$ is about 70 per cent. larger than for $\overset{-}{I}$.

With the exception of NaCl, which was taken as a starting point in calculat-

[1] W. L. Bragg, Phil. Mag., Aug., 1920.
[2] Davey and Wick, PHYS. REV., March, 1920.
[3] A. W. Hull, PHYS. REV., Jan., 1921.
[4] Davey, PHYS. REV., May, 1920.
[5] I. Langmuir, Jour. Am. Chem. Soc., 1, pp. 911–913, 1919.

ing the wave-length of x-rays used, only data obtained in this laboratory were used in the calculations. The dimensions assigned to various crystals will be found to differ slightly from those published by Bragg and his co-workers. This is because our distances are all calculated from the x-ray data instead of being calculated from the measured density of the crystal and the mass and shape of the unit crystal. In this way our results are independent of the errors in the published data on density, caused by impurities, inclusions in the crystals, etc.—and are mutually consistent.

Most of the structures and dimensions of crystals given below have already been published.[1] Table I. gives the name, structure, and dimensions of crystals by which the size of ions may be directly calculated. Table II. shows that the values so obtained are consistent with the dimensions of other crystals formed from the same ions.

TABLE I.

KCl, — Simple cube of ions $d = 3.13$ A	$\overset{+}{K} = 1.56$ Å
	$\overset{-}{Cl} = 1.56$
RbBr, — Simple cube of ions $d = 3.465$°	$\overset{+}{Rb} = 1.73$
	$\overset{-}{Br} = 1.73$
CsI, — Centered cube of ions $d = 4.58$	$\overset{+}{Cs} = 1.98$
	$\overset{-}{I} = 1.98$
NaCl, — Simple cube of ions $d = 2.81$	$\overset{+}{Na} = 1.25$
$\overset{+}{Na} = NaCl - \overset{-}{Cl}$ $2.81 - 1.56$	
KF, — Simple cube of ions $d = 2.69$	$\overset{-}{F} = 1.13$
$\overset{+}{K} = KF - \overset{-}{K}$ $2.69 - 1.56$	

TABLE II.

NaF	$d = 2.34$	$\overset{+}{Na} + \overset{-}{F} = 1.25 + 1.13 = 2.38$	Error < 2 per cent.
NaBr	$d = 3.01$	$\overset{+}{Na} + \overset{-}{Br} = 1.25 + 1.73 = 2.98$	" 1 " "
NaI	$d = 3.25$	$\overset{+}{Na} + \overset{-}{B} = 1.25 + 1.98 = 3.23$	" < 1 " "
KBr	$d = 3.30$	$\overset{+}{K} + \overset{-}{Br} = 1.56 + 1.73 = 3.29$	" < 1 " "
KI	$d = 3.55$	$\overset{+}{K} + \overset{-}{I} = 1.56 + 1.98 = 3.54$	" < 1 " "
CsCl	$d = 3.54$	$\overset{+}{Cs} + \overset{-}{Cl} = 1.98 + 1.56 = 3.54$	" < 1 " "

[1] A. W. Hull, Proc. Am. Inst. Elec. Eng., 37, 1919. W. P. Davey, PHYS. REV., March, 1921. W. P. Davey and Frances G. Wick, PHYS. REV., March, 1921.

The first five crystals in Table II. are simple cubes of ions. The last is a body centered cube of ions d 4.10. It will be noted that F comes out smaller than Na. This is hard to explain at present.

In order to extend this work to ions having a valence of two it is desirable to make measurements on SrSe and BaTe, treating these results as in the case of KCl, RbBr and CsI, but we have so far been unable to obtain any of these compounds. CaS can not be used in this way owing to the great percentage differences in the nuclear charges. The same end might be obtained using the halides of ions having a valence of two, provided that two different halides of the same ion could be employed, one as a check on the other. In this case the difficulties are considerable. Some of these compounds are so hydroscopic as to dissolve completely during the process of preparing the crystal powder. Others tend to decompose when heated to drive off water of crystallization. Still others give patterns which we have been so far unable to interpret. A report on bivalent ions will therefore be deferred until these difficulties are sufficiently overcome.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY.

FORCES WITHIN A STATIC ATOM.

BY IRVING LANGMUIR.

IF in addition to the Coulomb forces between charged particles we assume the existence of another force (quantum force) equal to

$$F_q = \frac{1}{mr^3}\left(\frac{nh}{2\pi}\right)^2$$

acting between an electron and a nucleus, we find that a stationary electron is in stable equilibrium when its distance r from a nucleus is the same as the radius of a circular orbit corresponding to a stationary state in Bohr's theory. The total energy of the electron is also the same as that given by Bohr's theory. The frequency of oscillation about the position of equilibrium is identical with the frequency of revolution of the electron in the Bohr atom. Thus the Rydberg constant and the Balmer series can be deduced from this law of force without assuming moving electrons. It seems possible to construct satisfactory models of the helium atom and the hydrogen molecule on this basis by assuming that each electron acts towards the nucleus like a dipole acting on a charge while the quantum action between electrons is like that between dipoles. Thus the forces depend on the orientation of the electrons as well as on their distance. This view suggests that quantum relationships may have their origin in the structure of the electrons and each change in the quantum number associated with an electron corresponds to a change in the structure of the electron.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

The Effect of Roughening of Surface on the Dispersion of Cannon Ball in Falling Through Air.[1]

By F. C. Brown.

Dr. Duff and other eminent physicists in this country and abroad proposed the substitution of spherical bombs for the stream line bombs used during the war with the expectation of reducing irregularities of ballistic performance.

Experiments were carried out at Aberdeen Proving Ground, in coöperation with the Balloon Section of the Air Service, showing that the variability of flight of spherical bombs was several times as great as that of the best stream line form.

Further experiments were carried out to determine the effect of roughening of the surface of spheres on the dispersion constant. The dispersion constant was taken as the mean dispersion, divided by the altitude, and for convenience is expressed in angular measure.

The tests on the smooth spherical bombs were carried out with 8-inch iron spheres salvaged from the Civil War; and also some spherical German bombs were used in the experiments. Four of these were dropped simultaneously and the positions of impact were plotted. To determine the effect of roughening the 8-inch cannon balls were drilled so as to give the appearance, when reduced, of dimpled golf balls. The accompanying photograph shows the appearance of the spheres. The plots of the positions of the smooth and roughened cannon balls at impact are shown in the accompanying diagram. The angle of dispersion for each trial is indicated by θ.

The following tables give the dispersion of the smooth as well as the roughened spheres:

Smooth Spheres Dropped Below 2000 Feet.

Altitude	Tan θ
1400 feet	.0048
1400 "	.0189
1400 "	.0103
1100 "	.0052
1000 "	.0196
1000 "	.0129
1000 "	.0140
Mean	.0122 ± .002

Smooth Spheres Dropped at 2000 Feet and Above.

Altitude.	Tan θ
2700 feet	.0120 +
2000 "	.0074
2000 "	.0104
2000 "	.0215
2000 "	.0147
Mean	.013 ± .002

$\tan \theta = .0125$

$\theta = .72°$ average for all altitudes.

[1] Abstract of paper to be presented at the Washington Meeting of the American Physical Society.

Roughened Spheres Dropped Below 2000 Feet.

Altitude.	Tan θ
1700 feet	.0025
1400 "	.0046
1400 "	.0099
Mean	.0057 ± .0030
	θ = .33°

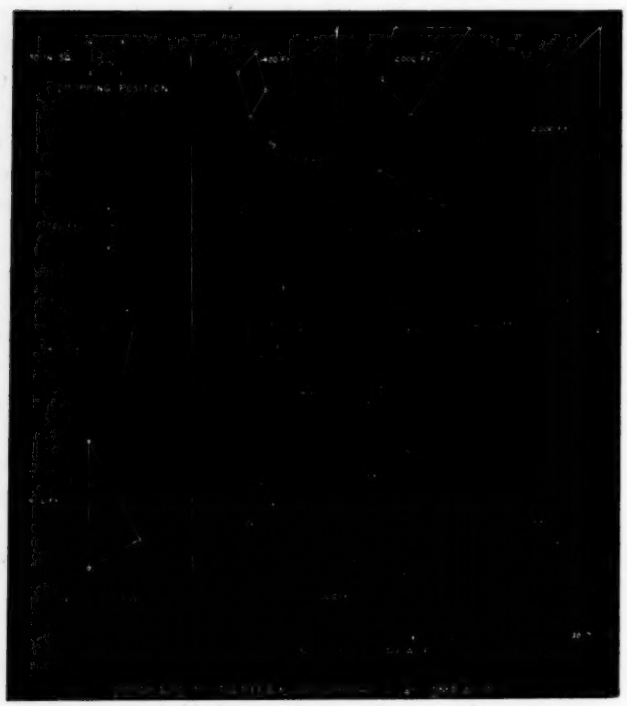

Fig. 1.

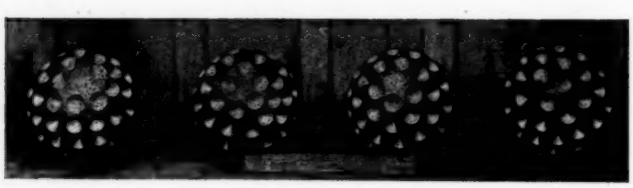

Fig. 2.

Even if we could roughen the surface of spherical bombs we would not then obtain as uniform ballistic properties as are obtained in the best stream line form.

The principal conclusion from this work is that the roughening of iron spheres reduced the dispersion constant to about one half.

BUREAU OF STANDARDS.

On the Determination of Instantaneous Pressures in Guns by Means of a Piezo-Electric Gage and a Ballistic Galvanometer.

By J. C. Karcher.

One or more plates of quartz which have been cut from a quartz crystal in such a way as to display the piezo-electric property are stacked up with suitable electrodes in the manner of a plate condenser. The plates are subjected to the gun pressure by means of a piston. The whole with suitable leads for removing the charge is assembled in a kind of capsule which screws into the breech block of a gun.

From the well-known Piezo-electric property of quartz one has the relation

$$q = Kp,$$

where p = the gun pressure at any instant, K is a constant of the gage and q is the quantity of charge liberated. This relation was found to hold for pressures up to 50,000 pounds per square inch. If the charge q is permitted to flow through a galvanometer the current

$$i = K\frac{dp}{dt}$$

and if θ is the angular displacement of the moving system of the galvanometer the following relation exists

$$iM = KM\frac{dp}{dt} = I\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + c\theta.$$

Since the pressure-time curve of a gun usually extends over a period of several hundredths of a second, a ballistic galvanometer may be used in which case the last two terms in above equation may be neglected and

$$\frac{dp}{dt} = \frac{I}{MK}\frac{d^2\theta}{dt^2},$$

$$p = \frac{I}{MK}\frac{d\theta}{dt} + c'. \quad (p = 0 \text{ when } \theta = 0 \therefore c' = 0.)$$

Displacement-time curves are obtained by photographing the galvanometer deflections on a uniformly moving film. The differential of this curve obtained either graphically or analytically is the time-pressure curve.

Bureau of Standards,
Washington, D. C.,
April 4, 1921.

A New General Law of Deformation.

By P. G. Nutting.

An investigation of pure shear in pitches possessing *both* elastic and viscous properties has led to the new general law of deformation s as a function of time t

$$s = at^n F^m$$

and stress F, in which n and m are independent of s, t and F, and of the units and method employed. For $n = 0$ and $m = 1$ this reduces to Hooke's law as a special case while for $n = 1$ and $m = 1$ it gives the law of viscous flow.

The corresponding forms for the electrical case

$$q = at^n X^m \quad \text{and} \quad i = ant^{n-1} X^m$$

give Ohm's law as a special case when $n = 1$ and $m = 1$. Data by Shrader on leakage through dielectrics is represented very well by the new law.

WESTINGHOUSE ELECTRIC AND MANUFACTURING CO.,
March 28, 1921.

CONTRACTIONS AND EXPANSIONS OF AMALGAMS WITH TIME.

BY ARTHUR W. GRAY.

THE dimensional changes that occur during the hardening of a dental amalgam are of such importance in connection with the tooth-restoring properties of this valuable filling material that determinations of these changes are nearly always included in tests of amalgam alloys. A contracting amalgam may shrink sufficiently to admit oral fluids and bacteria between the filling and the cavity walls; an expanding amalgam may swell sufficiently to extend above the margins of the cavity, or even to split the tooth. The ideal amalgam expands just enough to make sure that a properly inserted filling will remain firmly in contact with the tooth.

No theory yet advanced gives a satisfactory reason for the existence of these dimensional changes, which I call *reaction expansions* to differentiate them from *thermal expansions*. The prevailing belief is that silver causes expansion and tin contraction. Adjusting the composition of a dental alloy so that its amalgam will show a desirable reaction expansion is, therefore, regarded as "balancing" the opposing tendencies of the component metals.

A study of the modifications produced by systematically varying the conditions that affect reaction expansion has finally led me to formulate a simple theory that seems consistent with all facts at present known. I shall not enter into a consideration of this theory here, but shall merely present some of the experimental evidence upon which it is based.

Black had observed that some amalgams contracted a little before they expanded.[1] In a paper before the American Institute of Mining Engineers[2] I pointed out that the typical reaction expansion of a dental amalgam is characterized by four consecutive stages:

1. Rapid contraction to a minimum.
2. Somewhat slower expansion to a maximum.
3. Considerably slower contraction to a second minimum.
4. Very much slower expansion to a second maximum.

[1] G. V. Black, The Physical Properties of the Silver-Tin Amalgams, Dental Cosmos, 38, 982, 1896.

[2] A. W. Gray, Metallographic Phenomena Observed in Amalgams, Am. Inst. Min. and Met. Eng. Trans., 60, 684 and 693, 1919; Journ. National Dental Assn., 6, 913 and 918, 1910.

One or more of these stages can be masked by suitable mechanical treatment of the amalgam.

The gradual changes in shape that the reaction expansion curve can be made to undergo by systematic variation of influencing conditions are particularly well shown by a series of forty-two curves obtained during an investigation of effects produced by varying both the size of particle in which a dental alloy is cut and the pressure under which its amalgam is molded into a test specimen. For this investigation a large quantity of alloy filings from the same melt was sorted by standard sieves into five samples. The particles of the first sample passed a sieve with 100 meshes and were retained by one with 115 meshes to the inch. The particles of the fifth sample passed a 200-mesh sieve. Unsorted alloy, comprising a mixture of all sizes passing 100 meshes, furnished a sixth sample.

The reaction expansion at 37.5° C. was determined for seven specimens from each of the six samples of alloy. The packing pressures under which amalgam from each sample was molded were increased in geometric progression from 25 to 1,600 kg. per cir. cm. In order to obtain the required plasticity a greater mercury-alloy ratio was used with the finer samples than with the coarser ones, but otherwise packing pressure and size of alloy particles were the only factors that were varied.

Fig. 1 shows how variations in packing pressure affect the reaction expansion of amalgam from the sample of alloy that passed a 200-mesh sieve. The distance from one horizontal line to the next represents a change of 2.5 microns (1/10,000 inch) in the diameter of a freely expanding amalgam cylinder one centimeter in diameter, *i.e.*, a linear expansion of 0.025 per cent. Since an observation was taken every minute during the first seven hours after a cylinder was molded, the plotted points merge into a continuous line for each specimen. The change in 24 hrs. is indicated by a single dot at the right of the chart. Subsequent changes have been followed for several months and are still under observation.

The chart shows that each increase in packing pressure is accompanied by an increase in the rate of reaction (solution of the alloy in the mercury followed by crystallization of the amalgam), each prominent feature of the expansion making its appearance earlier than in the preceding curve. The influence of pressure change is very much greater at the lower pressures. For example, the amalgam packed under 25 kg. per cir. cm. requires about a month and a half to reach the first maximum. Doubling this pressure results in the maximum being reached during the first 24 hrs. Another doubling shortens the time to about 2.5 hrs. Additional increments of pressure progressively hasten the appearance of this maximum until under 1,600 kg. it is reached in less than ten minutes. At the same time, the increase of pressure progressively obliterates the initial rapid descent preceding the first minimum, and it finally obliterates some of the following rise towards the first maximum. It also hastens the appearance of the second minimum, which is reached in about three weeks with 50 kg. and in about three hours with 1,600 kg. With this

alloy the rise from the second minimum to the final maximum is negligible, being no more than one micron per centimeter, or 0.01 per cent.

Fig. 2 shows how variations in size of alloy particles affect the reaction expansion of amalgams packed under 25 kg. per cir. cm. Six more similar groups of curves have been charted, one for each of the other six pressures represented in Fig. 1. The figures to the right of each curve give the number of meshes per inch of the sieve that passed and the sieve that retained the alloy used for the amalgam represented by the curve. The curve at the top of the chart was obtained from the unsorted dental alloy, that is to say, from alloy in which particles of all the various sizes represented by the other curves were mixed in proportions that yielded a commercial product of the highest grade.

The control of the reaction expansion produced by this blending of various sized particles is evident on comparing the curve of specimen R464 with the other curves on the same chart, especially when the comparison is extended to include observations made at intervals during a period of several months. For example, specimen R467 expanded 0.18 per cent. beyond its first minimum, or 0.11 per cent. beyond its diameter when observations began a few minutes after molding, before it attained its first maximum a month and a half later. The other amalgams from the sorted particles of alloy behaved very much like this, although gradual changes in the expansion curve are noticeable as the alloy becomes progressively coarser.

In contrast with the specimens from sorted particles, R464 attains its first maximum in a few hours, with an expansion of less than 0.05 per cent. beyond its first minimum, or about 0.03 per cent. beyond its initial diameter. Contraction of about 0.02 per cent. to the second minimum, and final expansion too small for reliable measurement, leave the amalgam about 0.02 per cent. larger than when first measured. The same amalgam when packed under 50 kg. gives an expansion curve that is practically the same in all respects as the curve of R464.

The limitation of dimensional changes exhibited by the curve of R464 is not an accident of experimental procedure. It is a direct consequence of blending alloy particles that differ in size; this is shown by many almost identical curves obtained during routine tests of different lots of similarly made alloy.

The charts not reproduced in this paper show that, as the packing pressure is increased, the effect of variation in size of alloy particles becomes progressively less.

The contraction found when amalgam from a properly adjusted dental alloy is packed under a very high pressure does not in any way prevent the making of a tight tooth filling, because packing hard enough to cause contraction in such an amalgam will stretch the resilient dentin more than enough to make it follow the slight shrinkage of the filling. In fact, moderate contraction after very tight packing is an advantage, in that it relieves to some extent the straining of the tooth. The tighter the packing, the better the filling, because heavy packing pressure not only adds to the strength of the amalgam,

but also shortens considerably the time required for it to complete all its dimensional changes and become stable. Moreover, it secures much better adaptation of the filling to the cavity walls and, consequently, reduces liability to leakage.

The packing pressures used in this investigation extend from well within the dental range to considerably beyond it. The mean effective packing pressures employed by different individuals, in condensing amalgam fillings with dental instruments, vary widely. I found the average for four individuals to be 68 kg. per cir. cm. The average deviation from this was 64 per cent.; the difference between the highest (234 kg.) and the lowest (17 kg.) was 321 per cent.[1] Such deviations taken in conjunction with the pressure effects shown in Fig. 1 indicate that, regardless of the accuracy of the particular device employed for measuring the dimensional changes, too much reliance must not be placed on comparisons of dental alloys that are based on expansion tests of amalgams packed by hand pressure or by mallet blows.

Control of temperature is also important. Since the linear thermal expansivity of a dental amalgam is about 0.000025 per degree centigrade, a tempera-

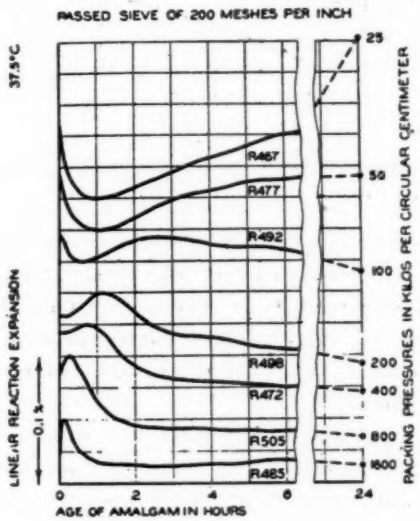

Fig. 1.

Influence of packing pressure on the reaction expansion of a dental amalgam made from an alloy containing silver, tin, copper and zinc. Approximately 69 per cent. silver.

ture variation of one degree will change the length of a one-centimeter specimen 0.25 micron. Results obtained at uncontrolled room temperatures with an interferometer sensitive to 0.005 micron are, therefore, not nearly so reliable as results obtained with a less but sufficiently sensitive instrument and accurate

[1] See reply to discussion of my paper entitled "Transition Phenomena in Amalgams," presented at the Columbus meeting of the American Institute of Mining and Metallurgical Engineers, October, 1920.

control of temperature. The dilatometer used for obtaining the curves of Figs. 1 and 2 is accurate to 0.05 micron; the temperature of the specimen during measurements seldom varies 0.01° from 37.5° C., or whatever other temperature is selected for the measurements.

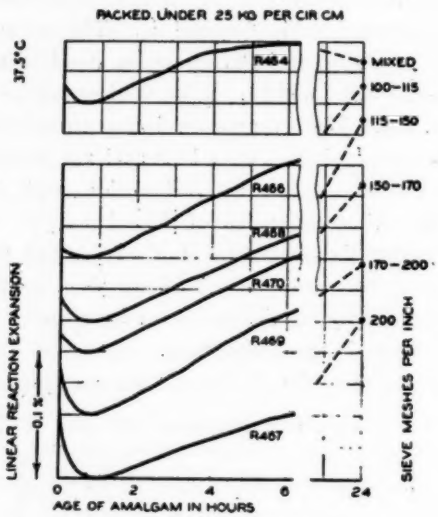

Fig. 2.

Influence of size of alloy particles on reaction expansion. Amalgam from alloy of same composition as in Fig. 1.

The reaction expansions of over five hundred amalgam specimens have been measured under accurately controlled conditions that have been systematically varied. Some specimens have been observed at regular intervals for about two years. In every case the expansion curve was found to conform to the behavior outlined above. Many examples were obtained of all four stages showing in the same curve.

From among the many interesting cases that might be cited as examples, only two will now be mentioned to illustrate long continued dimensional changes of considerable magnitude. Both specimens were prepared from commercial dental alloys by amalgamating according to the directions furnished by the manufacturers and packing under 50 kg. per cir. cm. One specimen, made from a coarse-grained non-zinc alloy, took more than a year to reach its first maximum. During this period it expanded as much as 0.48 per cent. The other, from an alloy containing zinc, reached its first minimum in half an hour, after contracting about 0.04 per cent. It then expanded 0.01 per cent. to its first maximum, which was reached within three hours after molding. The second minimum was reached in about a day, after a contraction of 0.02 per cent. below the first minimum, or 0.06 per cent. below the diameter when observations began. After this, the amalgam steadily expanded towards its second maximum, which was not reached in eight months, when observations

had to be discontinued because the cylinder had grown so large that readjustment of the dilatometer would have been necessary for further measurement. The diameter of the specimen had increased 1.28 per cent. and was still expanding. This abnormal expansion is not attributable to the zinc contained in the alloy.

The phenomena described in this paper are just what ought to be expected from a consideration of conditions that influence diffusion, solution, and crystallization. Accurate measurements of the dimensional changes that I have termed *reaction expansions* ought to throw light on other problems of metallography and physical chemistry.

PHYSICAL RESEARCH LABORATORY,
THE L. D. CAULK COMPANY,
MILFORD, DEL.,
April 5, 1921.

MEASUREMENT OF MERCURY VAPOR PRESSURE FROM 0° to 35° C. BY THE KNUDSEN PRESSURE GAUGE.

BY C. F. HILL.

NINETEEN values for the vapor pressure of mercury at various temperatures from $-.7°$ to 34.9° C. have been made by means of the Knudsen pressure gauge, calibrated against a McLeod gauge. These values all lie within about ± 6 per cent. of the mean curve, most of them lying within ± 2 per cent. The value at 0° is .000350, at 10° is .000775, at 20° is .00182, at 30° is .00407, and extrapolated to 40° gives .00787 mm. mercury. This last value agrees closely with the extrapolated values of Ramsey and Young. The slope of the curve also agrees with that of Ramsey and Young, and agrees fairly closely with that of Morley. The results of the present method indicate that the values given by Knudsen in 1909, are much too low, probably as much as 50 per cent.

The complete account of these measurements will be published soon.

UNIVERSITY OF ILLINOIS,
URBANA, ILLINOIS,
April 1, 1921.

THERMAL CONDUCTIVITY OF WHITE MARBLE AND "NEAT" CEMENT.

BY ROY A. NELSON.

THE thermal conductivity of white marble has been determined by a number of investigators. The principal method used has been the "plate" or "wall" method. It was thought that it might be of value to determine the thermal conductivity of this material by the "cylinder" method both for a check on this method of measuring the thermal conductivity of poor conductors and to give another value of the conductivity of white marble, which is of fairly definite composition.

The sample of marble tested was a very good grade of marble with a fine

grained structure. It was called "Alabama White" marble and chemical analysis showed that it was principally calcium carbonate with a small amount of magnesium carbonate. In order to be sure that the marble was free from moisture it was heated in a large oven at 130° C. for four hours.

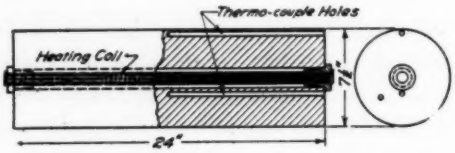

Fig. 1.

The figure shows sections of the cylinders. The central hole for inserting the heating coil was 3.5 cm. in diameter and the holes for inserting the thermocouples were 0.4 cm. in diameter. Cylinders made of Portland cement and water were also tested for the thermal conductivity.

The diffusivity or the thermometric conductivity was calculated for these materials. This was done by determining the densities and the specific heats as well as the conductivities.

Perhaps the most complete investigation on the conductivities of marbles is that of Peirce and Wilson.[1]

The properties of the "Alabama White" marble tested correspond very closely to the marble tested by the above authors, called "American White." The values determined in this investigation on marble and neat cement as well as those given by Peirce and Wilson for "American White" marble are given below:

	"Alabama White" Marble.	"American White" Marble.	"Neat" Cement.
Density	2.71	2.72	1.83
Specific heat	.213	.214	.278
Conductivity	.00614	.00596	.0014
Diffusivity	.0106	.0102	.0275

A marked decrease in the conductivity of the marble with increasing temperature between 50 and 200° C. was found. For the neat cement no pronounced change in the conductivity with the temperature was found. The values obtained for the conductivities at different temperature ranges are given below. Neither Peirce and Wilson nor others so far as we have found

	50° C. to 100° C.	100° C. to 150° C.	150° C. to 200° C.
Marble	.00614	.00524	.00415
"Neat" Cement	.00140	.00165	.00146

[1] Proc. Am. Acad., Vol. 34 and 36, 1898 and 1900.

in the literature, give values of the thermal conductivity of marble for temperatures over 100° C. We were unable to determine the conductivity at temperatures above 200° C. owing to the fact that the cylinders cracked at higher temperatures.

The above determinations were made in connection with an extended investigation by Professor A. P. Carman and the writer on the thermal conductivity and diffusivity of concrete mixtures, to be published shortly by the University of Illinois Engineering Experiment Station.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
March, 1921.

THE EFFECT OF PRESSURE ON THE THERMAL CONDUCTIVITY OF METALS.

BY P. W. BRIDGMAN.

TWO methods of measurement have been employed; the first is a radial flow method in which the heat is put in at the axis of a cylinder of the metal, and the difference of temperature determined at two different radial distances. The second method is a longitudinal flow method, in which the correction for the lateral loss of heat to the medium which transmits pressure is kept low by cutting down the dimensions of the specimens.

Measurements to 12,000 kg. per sq. cm. have been made on the following metals: Lead, tin, cadmium, iron, copper, nickel, silver, platinum, bismuth, antimony and zinc.

Within the limits of error the effect is linear with pressure. Of the eleven metals above, the thermal conductivity of four is increased. These four are: lead, tin, cadmium, and zinc. The thermal conductivity of the other seven metals decreases under pressure; the decrease of iron is least and that of bismuth the greatest. The increase of thermal conductivity of lead and tin is greater than the increase of electrical conductivity, so that the pressure coefficient of the Wiedemann-Franz ratio of these two metals is positive. The pressure coefficient of the Wiedemann-Franz ratio of the other nine metals is negative.

HARVARD UNIVERSITY.

THE NEW FIXED POINT ON THE THERMOMETRIC SCALE.

BY FREDERICK BATES AND F. P. PHELPS.

WHEN crystalline quartz is heated, recrystallization occurs in the neighborhood of 570° C.–575° C. In studying the rotation of the plane of polarization at high temperatures in this material, it was discovered that the energy changes occurred with great suddenness at the critical point when the crystal was absorbing energy. Careful exploration of this region with a precision thermocouple showed that the absorption of energy by a plate of quartz as small as 10 mm. in diameter by 1.5 mm. in thickness could readily be detected with the thermo-

couple junction placed in the immediate vicinity of the plate. The heating and cooling curves of larger plates were obtained by drilling an opening in the plate sufficiently large to insert the thermocouple. It was found that the inversion begins with great suddenness at 573°.3 C. Repeated determinations at this point showed a maximum variation of but 0.2° C. for a large number of determinations, and the experimental error for repeated determinations, using the same plate of quartz, was found to be of equal magnitude with that obtained using quartz from various parts of the world.

In order to standardize a thermocouple for this portion of the temperature scale, it is only necessary to take any piece of clear crystalline quartz, insert the thermocouple in the small opening therein, and heat slowly. Upon plotting voltage against time, an extremely sharp break at 573°.3 C. will be found.

In view of the attempts of previous investigators to definitely determine the temperature of inversion from alpha to beta quartz, it can now be definitely stated that the beginning of the change, as shown by the heating and cooling curves, are the only definitely reproducible points in the transformation.

BUREAU OF STANDARDS.

A DISAPPEARING FILAMENT OPTICAL PYROMETER FREE FROM DIFFRACTION EFFECTS AT THE FILAMENT.

BY C. O. FAIRCHILD.

THE writer has been using for the past two years an optical pyrometer in which the filament of the pyrometer lamp could be made to disappear perfectly at any magnification and resolving power of the eyepiece. With this instrument the precision of photometric measurement has been somewhat increased over that of any other form of photometer.

In order to obtain these results it was necessary to use such apertures in the telescope as would eliminate so far as the eye could detect the effects of diffraction at the pyrometer lamp filament. Proper attention to the laws of diffraction and the nature of optical images, together with experiment have made it possible to state definitely what the relation of these apertures to one another should be. The conclusion arrived at by Worthing and Forsythe[1] that in general the brightness of the pyrometer filament is not equal to the brightness of the background image when a photometric match is obtained is found applicable to special cases only. These writers made a study of the effects of diffraction at the pyrometer filament but evidently confused the effects of the filament and the diaphragms. Limitations which they placed on the design and use of the pyrometer are in nearly all cases removed.

In Fig. 1 stops *A* and *B* subtend equal angles at the center of the image *I*. In order that this image should be of uniform brightness geometrical optics would require that the aperture be not smaller than *C*. From diffraction considerations this is not, however, a sufficient condition, if an obstacle partly screens *I*. The latter conclusion can be derived directly from the classic

[1] Worthing and Forsythe, PHYS. REV., 4, No. 3, Sept., 1914.

diffraction theory of optical images and the effects of small circular apertures. The line AB in Fig. 2 bounds the shadow of the lamp filament F. Light is

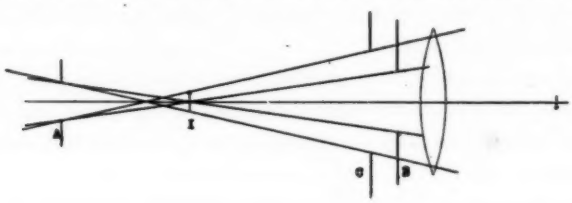

Fig. 1.

diffracted into this shadow. This light is supplied by that coming from a few zones BD contiguous to B and to a negligible amount by the light entering the central portion of the aperture BC. If the exit stop reaches close to the

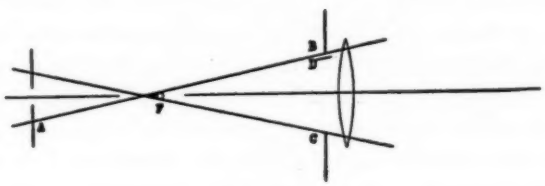

Fig. 2.

point A interference fringes will be visible. But if this exit aperture is sufficiently small the fringes near A will be screened from view. That is, the edges of the pyrometer filament will be sharply outlined against the background.[1]

Within the shadow of the filament, light passing the two edges will interfere and for small filaments (1 to 4 mils) a single bright fringe on the axis is visible when the filament is unheated and the background image is brilliant. This fringe is most intense for small apertures and is only eliminated by relatively large apertures. It does not however effect the brightness match if the magnification is sufficient and the resolution good enough to prevent the observer from averaging this brightness with the remainder of the filament. With small apertures this fringe is broadened and can be made to cover the entire filament. In no case is it visible when the filament is matched in brightness with the background.

It follows then that the entrance angle of the pyrometer telescope must be so much larger than the exit angle, that diffraction caused by it shall produce fringes totally screened from view by the stop of the exit. In other words the image of the background, itself a diffraction pattern, must be sufficiently complete that larger entrance angles, *i.e.*, the addition of zones at the aperture periphery will produce a negligible change in this pattern.

A pyrometer with aperture selected as above can be used for sighting on small

[1] This aperture must be large enough to allow good resolution and will depend somewhat on the magnification. Photographs have been made to illustrate this.

objects such as lamp filaments or openings of experimental "black body" radiators, without undue magnification, for it is found that the size of the image plays no part in the above treatment until it is nearly completely hidden by the filament of the pyrometer lamp.

Brightness measurements have been made proving that within experimental error the image and pyrometer lamp filament can be matched when they are truly of equal brightness, and this through wide ranges of magnification and aperture values.

The proper selection of apertures is accomplished with ease by using a good eyepiece and an iris diaphragm in the telescope objective. The entrance angle is increased until the filament can be made to disappear. In one case this occurred at an angle of .03 radians with an exit angle of .013 radians. This proportion does not, of course, persist.

A mathematical treatment of this subject would be very difficult. Mathematicians have usually limited their solutions of diffraction problems to plane waves, point sources, or to very small apertures.[1] Observations in experiment have likewise been so limited.[2] Little attention seems to have been paid to the phenomena as modified by converging rays and by screening off a portion of an image by a screen in the focal plane.

Basu found the interference fringes in his experiments depending on the diameter of the filament. He ascribed these variations to the interference of the light reflected from the obstacle with the direct grazing rays. He observed as many as twenty fringes, using a point source. The writer has counted twelve fringes using illumination by converging rays from a fairly large source. These fringes are very sharp and very close to the filament.

That such fringes may be screened from view on the axis of the optical arrangement is apparently a new observation.

BUREAU OF STANDARDS, WASHINGTON, D. C.,
April 5, 1921.

THE ORGAN PIPE AS A COUPLED SYSTEM.

BY ARTHUR TABER JONES.

THE various changes in the pitch of an organ pipe as the pressure is gradually increased have never been fully explained. Lunn[3] has considered the small change in pitch when the pressure is near that for which the pipe is voiced. The changes in pitch have been examined experimentally by various observers, notably by Wachsmuth[4] and his pupils. Our understanding of the reed pipe has recently been considerably advanced by Vogel,[5] who has treated the reed pipe as a coupled system. The reed alone has a certain natural frequency, the

[1] See for example works of Sommerfeld, Lommel, Hargreaves, Lord Rayleigh.

[2] On the Diffraction of Light by Cylinders of Large Radius, Nalinimohan Basu, Phil. Mag., 35, p. 79, Jan., 1918.

[3] PHYS. REV., (2), 15, p. 446, 1920.

[4] Ann. der Physik, (4), 14, p. 469, 1904.

[5] Ann. der Physik, (4), 62, p. 247, 1920.

tube above it has a series of natural frequencies, and the tones from the reed and the tube react upon each other. It seemed possible that the flue pipe might be regarded in a similar manner. The tube alone has a series of natural frequencies, the wind striking the upper lip of the pipe produces an edge tone, and the tones from the tube and the upper lip react upon each other. The pitch of the edge tone depends upon the blowing pressure, and various observers[1] have found that this pitch rises with increasing pressure, that the pitch is usually discontinuous at certain points, and that similar statements hold for organ pipes.

If a flue pipe may be regarded as a coupled system we should expect the frequency of the sound from the pipe to be in the neighborhood of the natural frequency of that component which is the less damped. Now although the radiation of the sound from the pipe causes a rather large damping of the component from the tube, it seems likely that the edge tone is much more strongly damped. We should therefore expect the frequency of the sound from the pipe to be not far from the natural frequency of the tube, and we should also expect it to deviate from that frequency most widely when the natural frequency of the edge tone is near that of the tube. A preliminary study appears to substantiate this expectation.

The pipe used is a wooden open diapason built by the Austin Organ Company. The tube is 67.6 cm. x 9.6 cm. x 7.1 cm. inside. The pipe is voiced for a pressure of 3 in. [= 7.6 cm.] of water, and the upper lip is 19 mm. above the slit. All pitches were determined by comparison with a sonometer. The natural frequency of the tube was found by sending a puff of air quickly toward the mouth of the pipe from outside, and catching the pitch as the sound died away. The natural frequency of the edge tone was found by stuffing the upper part of the pipe with cotton batting, and then blowing the pipe in the usual manner. In Fig. 1 the frequency of the note from the pipe when no

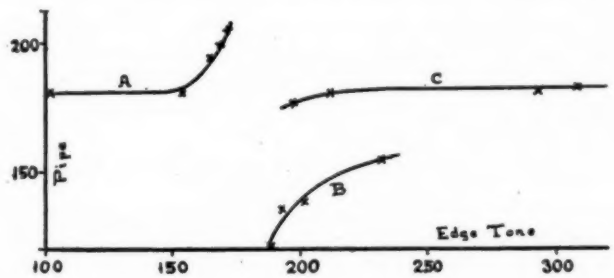

Fig. 1.

batting was used is plotted as a function of the corresponding frequency of the edge tone. For the curves *A* and *B* the pitch of the edge tone was so far below the natural pitch of the tube that the octave of the edge tone, which could usually be detected with a resonator, has been used instead of the funda-

[1] See Krüger, Ann. der Physik, (4), 62, p. 673, 1920.

mental. The ranges of pressure for the curves are as follows: *A*, 3.1 mm. of water to 4.9 mm.; *B*, 8.9 mm. to 11.1 mm.; *C*, above 23 mm. The curve *C* slopes slowly upward to the right to the highest pressure that was available (20 cm. of water). This pressure was not sufficient to make the pipe jump to the octave.

The natural frequency of the tube is about 185 vd. It is seen that if the pipe may be treated as a coupled system, and if the coupling is rather close, the curves are of the general form that is to be expected.

SMITH COLLEGE.

ON THE MEASUREMENT OF THE ACOUSTIC OUTPUT AND EFFICIENCY OF FOG-ALARM APPARATUS.

BY LOUIS V. KING.

AN account is given of fog-alarm tests carried out at Father Point, Que., in September, 1917.

When sound is being generated by compressed air, there is a considerable drop in temperature represented by the mechanical work propagated away as sound.

A small diaphone was installed and operated to give a continuous blast, so that this temperature drop, which is used in measuring acoustic output, could be studied. The results of the 1913 tests were confirmed, and it was found possible to obtain the acoustic characteristics of the apparatus by the use of two thermo-elements, one of which was inserted on the high-pressure side, the other on the low pressure side of the diaphone.

Combining these results with the harmonic analysis of the waves by Professor D. C. Miller it is now possible to state the amount of power contained in the fundamental and in each overtone.

Some evidence was obtained which showed that the harmonics were gradually killed out by propagation in the free atmosphere, the fundamental alone surviving at a distance of three miles or more.

MCGILL UNIVERSITY,
MONTREAL.

A SIMPLE APPARATUS FOR TESTING PITCH CONTROL.

BY PRESTON EDWARDS.

THE unit used in this apparatus is a cylindrical resonator to the face of which is attached a light torsional vibrator, tuned to the same pitch as the resonator. The vane of the vibrator is placed just in front of the mouth of the resonator, so as to be subjected to the reinforced vibrations of the air in that region. To the vibrator is attached a light concave mirror, and the resonator is so mounted as to permit of vertical and horizontal adjustment of the image. A number of these units are used, tuned at intervals of about one eighth of a tone, so that five will cover a semitone or nine a whole tone. The images are

brought into a line on the screen, in the order of pitch, and form a good means of testing, by the broadening of one or more of them, whether a singer or performer can hold true pitch. It can also be used, by taking oscillograms, to determine the true nature of trills, vibratos, tremolos, etc.

JOHNS HOPKINS UNIVERSITY.

SOME NEW CONSEQUENCES OF THE THEORY OF ULTIMATE RATIONAL UNITS.

BY GILBERT N. LEWIS.

LEWIS AND ADAMS[1] stated the theory of ultimate rational units. From this new theory they were able to obtain exact expressions for the constant of Stefan's law and for the Planck constant h. Later experimental determinations of these quantities have given results in remarkable agreement with their calculations.

With the centimeter as unit of length, a unit of time may be chosen which makes the velocity of light unity. If desired, this unit may also be called the centimeter, and we may say with Minkowski that 1 sec. $= 2.999 \times 10^{10}$ cm. Adopting this system, we still have two independent units, length, and mass or energy. According to Lewis and Adams, there is a fundamental constant in nature which has the dimensions of energy $\times$ length. It is $(4\pi\epsilon)^2$, where ϵ is the electron charge in electrostatic units. By choosing the unit of energy so that this constant becomes unity and calling this new unit the reciprocal centimeter, we may write 1 erg $= 2.778 \times 10^{16}$ $(\text{cm.})^{-1}$, or, in other words, the erg-centimeter is equal to 2.778×10^{16} ultimate rational units. It is in such units that the constant of Stefan's law (energy-density of radiation proportional to T^4) becomes unity.

According to an equation of Sackur, the entropy of a monatomic gas is given by the expression

$$S = R \ln T^{3/2} V M^{3/2} + K_1 = R \ln K_2 T^{3/2} V M^{3/2},$$

where T is the absolute temperature, V the volume and M the molecular weight. $K_1 = R \ln K_2$ is a constant of nature. According to the theory of ultimate rational units, if M is the mass of one molecule and V is the volume per molecule, then in the new units $K_1 = 0$ and $K_2 = 1$. Working back from this equation, the entropy of a monatomic gas can be calculated with a hundred-fold greater accuracy than it can be measured. The results are in complete agreement with existing data within the limits of their experimental error.

UNIVERSITY OF CALIFORNIA.

PRESENT STATUS OF THE ELECTRIC AND MAGNETIC UNITS.

BY J. H. DELLINGER.

IT is becoming more generally recognized that the units used in practically all electric and magnetic measurements are those of the international system.

[1] PHYS. REV., 3, 92, 1914.

TABLE I.

The International System of Electric and Magnetic Units.

Quantity.	Symbol.	Unit.	Defining Equation.	Dimensional Formula.
Resistance.....	R	Ohm		$[R]$
Current.....	I	Ampere		$[I]$
Electromotive force.....	E	Volt	$E = RI$	$[E] = [RI]$
Quantity of electricity.....	Q	Coulomb	$Q = \int Idt$	$[Q] = [IT]$
Electrostatic capacity.....	C	Farad	$C = \frac{Q}{E}$	$[C] = \left[\frac{T}{R}\right]$
Dielectric constant[1].....	K		$C = \frac{1.1124}{4\pi(10)^{12}} \frac{KS}{l}$	$[K] = \left[\frac{T}{LR}\right]$
Electric field intensity.....	$\mathfrak{E}$	Volt per cm.	$\mathbf{E} = \frac{dE}{ds}$	$[E] = \left[\frac{RI}{L}\right]$
Electric displacement....	D	Coulomb per cm^2.	$D = \frac{1.1124}{4\pi(10)^{12}} K\mathbf{E}$	$[D] = \left[\frac{IT}{L^2}\right]$
Self-inductance[2].	$\mathfrak{L}$	Henry	$\mathbf{L} = \frac{E_1}{dI_1/dt}$	$[\mathbf{L}] = [\mathbf{M}] = [RT]$
Mutual inductance[2].....	$\mathfrak{M}$	Henry	$\mathbf{M} = \frac{E_2}{dI_1/dt}$	
Magnetomotive force.....	$\mathfrak{F}$	Gilbert	$\mathbf{F} = \frac{4\pi}{10} NI$	$[\mathbf{F}] = [I]$
Magnetic field intensity (or magnetizing force).....	H	Gilbert per cm.	$H = \frac{d\mathbf{F}}{ds}$	$[H] = \left[\frac{I}{L}\right]$
Magnetic flux[4]..	ϕ	Maxwell	$\phi = 10^8 \int Edt$	$[\phi] = [RIT]$
Magnetic flux density.....	B	Gauss	$B = \frac{d\phi}{dS}$	$[B] = \left[\frac{RIT}{L^2}\right]$
Permeability....	μ		$B = \mu H$	$[\mu] = \left[\frac{RT}{L}\right]$
Reluctance.....	$\mathfrak{R}$	Oersted	$\mathbf{R} = \frac{\mathbf{F}}{\phi}$	$[\mathbf{R}] = \left[\frac{1}{RT}\right]$
Intensity of magnetization	J	.....	$B = B_0 + 4\pi J$	$[J] = \left[\frac{RIT}{L^2}\right]$
Magnetic susceptibility	κ	.....	$J = \kappa H$	$[\kappa] = \left[\frac{RT}{L}\right]$
Magnetic pole strength[5].....	m	.....	$\phi = 4\pi m$	$[m] = [R\ IT]$

[1] The defining equation is the formula for a parallel plate condenser of large S. This equation and other equations containing K differ from the corresponding equations used with c.g.s. units, by the factor $1.1124(10)^{-12}$; this factor is of historical origin, and equals $10^9/c^2$ where $c = 2.9982(10)^{10}$. In the author's paper, Bureau of Standards S292, of five years ago

Every complete system of electric units requires four fundamental units. In the international system of electric units these are the international ohm, the international ampere, the centimeter, and the second. This system is by no means the same as the so-called "practical" electromagnetic system. A few of the "practical" electromagnetic units were the historical basis from which the corresponding international units were developed, but the common statement that electric measurements are ordinarily made in the "practical" electromagnetic units, is erroneous. Many of the units in the "practical" electromagnetic system are of entirely inconvenient magnitude and are never used; for instance, the "practical" unit of magnetic flux density is 10^{-10} of the gauss. The international system, on the other hand, has been built up in such a way that the units are of convenient size and are the ones actually used.

Three of the four fundamental units of the international system have been defined by international agreement in terms of concrete standards. These are the international ohm, ampere and centimeter. The standards, in terms of which these units are defined, are maintained by the national standardizing laboratories and the International Bureau of Weights and Measures. The fourth fundamental unit, the second, of course makes use of the rotating earth as a standard.

Starting from these four fundamental units a complete system of electric and magnetic units is built up, using the equations of ordinary electric and magnetic practice, particularly those used in establishing standards. The resulting units are all of convenient magnitude and are the units ordinarily used, except of course, that the farad is too large for most purposes, and instead the microfarad and micromicrofarad are usually used. The units, defining equations, and dimensional formulas in terms of R, I, L and T, are shown in the preceding table.

The unit of energy in this system is the joule and the unit of power the watt. This system has the great advantage that both K and μ of empty space = 1. No other system of electric units has this advantage except the Gaussian and the Heaviside c.g.s. systems, which suffer from the disadvantage that every equation containing both electric and magnetic units contains the factor c, which = $2.9982(10)^{10}$.

The following units have been definitely adopted and defined by international

a unit of K was defined which eliminated this factor. Practice, however, has favored the unit and equations here presented.

[3] In the definition of self-inductance, E_1 is the e.m.f. created in a circuit by the variation of the current I_1 in the same circuit.

[3] In the definition of mutual inductance, E_2 is the e.m.f. induced in a circuit by the variation of the current I_1 in another circuit.

[4] Φ in the equation is the change of flux in the time t giving rise to the e.m.f. E in a closed circuit linking with the flux.

[5] Φ in this equation is the total magnetic flux emanating from or associated with magnetic poles of aggregate pole strength m; it is strictly equal to the resultant flux through the magnet only when the magnet is very long and thin.

agreement: the international ohm, ampere, volt, coulomb, farad, and henry. The units of the strictly magnetic quantities do not as yet have such definite and official status. Those given in the table are the units adopted and used by the American Institute of Electrical Engineers for many years past. Only one has actually been adopted by international action, the maxwell, adopted by the 1900 Paris electrical congress. The A. I. E. E. Standards Committee has just this month decided to press for international adoption of this complete set of magnetic units.

The international units as given in Table I are supplemented in engineering practice by an alternative set of magnetic units in which the gilbert is replaced by the ampere-turn. These units are most convenient for magnetic engineering calculations, and at some future time it may become desirable to give them international status as a set of units supplementary to the ordinary international units. Those of the ampere-turn units which differ from the ordinary units are shown in Table II. below.

TABLE II.

The Ampere-Turn Units.

Quantity.	Symbol	Unit.	Defining Equation.	Ordinary Units in one Ampere-turn Unit.
Magnetomotive force.....	$\mathfrak{F}$	Ampere-turn	$\mathbf{F} = NI$	$\frac{4\pi}{10}$
Magnetic field intensity (or magnetizing force).....	H	Ampere-turn per cm.	$H = \frac{d\mathbf{F}}{ds}$	$\frac{4\pi}{10}$
Reluctance.....	$\mathfrak{R}$	Ampere-turn per maxwell	$\mathbf{R} = \frac{\mathbf{F}}{\phi}$	$\frac{4\pi}{10}$
Intensity of magnetization	J	Maxwell per cm².	$B = B_0 + J$	$\frac{1}{4\pi}$
Magnetic susceptibility	κ	.....	$J = \frac{4\pi}{10}\ \kappa H$	$\frac{1}{4\pi}$
Magnetic pole strength.....	m	Maxwell	$\phi = m$	$\frac{1}{4\pi}$

The ampere-turn units are "rational" in the Heaviside sense, that is, the 4π's are redistributed in the equations involving these units. Rationalization is attained without changing the unit of permeability, a vital superiority of the ampere-turn units over the various systems of rationalized systems that have been provided except the Heaviside system. The ampere-turn units are moreover superior to the Heaviside in that the 4π's are more advantageously distributed and the units are the ones in actual use in electrical and magnetic measurements. It is remarkable that a set of units which has been evolved in ordinary practice is more satisfactorily rationalized than any of the many

systems that have been constructed from theoretical considerations with that very purpose in view.

Bureau of Standards,
Washington, D. C.

Discussion of a Kinetic Theory of Gravitation II., and Some New Experiments in Gravitation.

By Charles F. Brush.

Kinetic Theory of Gravitation.—The author's original (1910) theory is briefly reviewed, viz., that gravitation is due to the intrinsic energy of the ether, which is assumed to exist in wave form or energy flux of some kind propagated with high velocity in all conceivable directions so that the ethereal energy is isotropic, and is uniformly distributed everywhere except as modified by the presence of matter. The interblending energy shadows between gravitating bodies constitute a region of less than normal energy density into which the bodies are pushed by the superior energy density beyond them. *It is again insisted* that the energy acquired by a falling body is derived from the ether.

Nature of Ethereal Energy.—The hypothesis is proposed that *the ether is abstract energy* unassociated with anything. If the quantum theory of energy is tenable, we may perhaps regard the ether as *a vast atmosphere of energy quanta in violent agitation.*

Diamagnetism May Modify the Gravitational Field of a Diamagnetic Substance such as Bismuth.—The very slight *negative* magnetic permeability of bismuth *may cause a slight reflection or scattering of the gravitational energy flux.* This thought prompted the first of the series of experiments following.

1. *Gravitational Attractions of a Kg. each of Bismuth and of Zinc for a Small Nearby Ball of Silver Are Not the Same.*—Apparatus such as used by C. V. Boys in finding the gravitational constant and mean density of the earth was employed with slight modifications. The bismuth and zinc were in the form of cylinders of same size and shape. The torsional deflection in each case was divided by the weight of metal causing it, so as to reduce both to a common basis of attraction per Kg. *Taking the attraction of zinc as* 100, *that of bismuth is only about* 72.

2. *A Gravity Pendulum With a Bismuth Bob Has a Shorter Period Than a Similar Pendulum of Exactly the Same Oscillation Length Having a Zinc Bob.*—Many carefully designed and conducted experiments appear to demonstrate this.

3. *A Torsion Pendulum Having Equal Horizontal Arms Loaded With Equal Weights of Bismuth Has a Shorter Period Than When The Arms are Loaded With Equal Weights of Zinc of The Same Weight as The Bismuth Loads; The Radii of Gyration Being Exactly The Same in Both Cases.*—Many careful experiments appear to demonstrate this.

The author does not claim that diamagnetism is the cause of the remarkable behavior of bismuth, but thinks it probable: And he hesitates to draw general

conclusions until after further experimentation on different lines now in progress. It seems fairly clear however that *weight* and *mass* are not so simply related as heretofore supposed.

CLEVELAND, OHIO,
April 4, 1921.

DEFECTS IN SYMMETRIC OPTICAL SYSTEMS.

BY IRWIN ROMAN.

AT the St. Louis meeting[1] it was shown that for a lens system composed of quadrics of revolution, the curvature of the field, the relative error in zonal magnification and the angular coma as seen from the diaphragm are independent of the eccentricity of the generating conic, while an explicit formula was given for the longitudinal aberration.

Since then it has been shown that if we neglect terms of orders beyond the second, every surface of revolution can be replaced by an equivalent quadric surface. Suppose that the surface is generated by revolving the curve

$$x = \chi^0 + \psi_1 y^2 + \psi_2 y^4 + \cdots, \qquad \psi_1 \neq \infty,$$

around the x-axis. The normal at the point whose optical height is h will then cut the axis at a point a distance r from the axial point of the surface and a distance s measured along the normal, where, neglecting terms of higher order n h, we have

$$r = s = \frac{1}{2\psi_1} + B\psi_1 h^2.$$

The quantity $B = 1 - \psi_2/\psi_1^3$ may be called a "coefficient of form."

For a sphere the value of B is zero, so that B is a measure of the departure from sphericity. If we generate the surface by revolving a conic around that axis which contains the foci, we get $B = e^2$. If we revolve the conic around that axis which does not contain the foci, we get $B = e^2/(e^2 - 1)$. If B is positive, there thus always exists an equivalent quadric of the former type, while if B is negative, or if it exceeds unity, there always exists an equivalent quadric of the latter type. The revised formula for longitudinal aberration merely replaces e^2 of the previous formula[1] by B.

THE TRANSMISSION OF WAVES THROUGH A SYMMETRIC OPTICAL INSTRUMENT.

BY IRWIN ROMAN.

IN the passage of a wave through an optical instrument, each point of the wave traverses a path which establishes a correspondence, point by point, between all the surfaces of the system, both lens and wave surfaces being considered as in the system. If each surface is expressed in terms of its own Gaussian parameters, such as are used in differential geometry, the parameters

[1] See this journal, March, 1920, p. 221.

for every surface can be expressed, at least theoretically, in terms of the parameters for an arbitrary incident wave.

If the system is one of revolution about a common axis, the azimuth around this axis may be selected as one of the two Gaussian parameters, so that the problem contains essentially only one parameter. This is the case considered in detail in the present work. The rectangular coördinates of each point of a surface are considered as expressed by power series in terms of the parameters selected, each surface having one parameter and its own set of coefficients in the series. The parameters are all reduced to terms of that for a particular incident wave. Certain invariant combinations are found among the coefficients of pairs of surfaces, these invariants furnishing the refracted surface for a particular incident wave and refracting surface, as well as the wave after propagation within a single medium.

While the treatment is one involving power series, the solution is complete in the sense that recursion formulas make it possible to retain terms of all orders, although explicit forms have been calculated for only the first four orders. As an application, the longitudinal aberration of a symmetric wave is calculated. For a particular instrument with four refracting surfaces the results obtained by this method and by the usual method, agree remarkably well. For the case of a plane wave refracted through a single spherical surface, the results agree with those found in the literature.

The method is applicable to non-spherical surfaces as easily as to spherical ones, provided that all surfaces of the system are symmetric with respect to the same axis. The detailed analysis for the unsymmetrical case is contemplated for the near future, but has not yet been started.

NORTHWESTERN UNIVERSITY.

A DIRECT READING SPECTROPHOTOMETER FOR MEASURING THE TRANSMISSIVITY OF LIQUIDS.

BY IRWIN G. PRIEST.

THIS spectrophotometer consists essentially of a constant deviation spectrometer (Hilger) in combination with the author's "exponential" or "variation of thickness" photometer first exhibited at the Convention of the American Oil Chemists' Society, New Orleans, May 11, 1920.[1] The essential features of the latter are:

(*a*) Two beams of light proceed horizontally, one above the other from a uniformly illuminated vertical surface.

(*b*) A rotating sector disk of known transmission can be interposed in the upper beam.

(*c*) By means of two totally reflecting partially immersed rhombs, the lower beam is diverted through a variable thickness of liquid determined by the distance between the rhombs, one of which is carried by a slide on a track parallel to the beams of light. The liquid is contained in a horizontal trough with open top into which the rhombs dip.

[1] Priest, Cotton Oil Press, July, 1920, p. 96.

(*d*) The two beams are brought into juxtaposition in the photometric field by means of an arrangement of biprisms.

(*e*) The thickness of liquid in the lower beam is varied until its transmittance equals the transmission of the sector disk as judged by equality of brightness in the photometric field.

(*f*) A suitable scale provides for direct reading of transmissivity or the logarithm of transmissivity.

Two methods of obtaining the divided photometric field have been used, namely:

(*a*) A biprism at the spectrometer slit gives rise to two juxtaposed spectra which are viewed by a positive ocular;

(*b*) A biprism over the objective of the spectrometer telescope gives a divided field viewed without an ocular.

The air-glass surfaces of the totally reflecting immersion rhombs are protected by glass covers fused to the rhombs by the method of Fairchild.[1] The author is greatly indebted to Mr. Fairchild for the preparation of these rhombs.

This instrument has been developed to meet the needs of *rapid* as well as accurate work. It is direct-reading in transmissivity, logarithm of transmissivity and wave-length. The instrument does its own computing automatically and a spectral curve may be determined and plotted in a few minutes. It is thought it will be especially useful in the technologic examination of oils, dye solutions, sugar solutions, etc.

The commercial manufacture of this instrument is being undertaken by Keuffel and Esser, Hoboken, N. J. A detailed description will be published later.

NATIONAL BUREAU OF STANDARDS,
April 6, 1921.

THE EXCITATION OF ENHANCED SPECTRA IN LOW VOLTAGE ARCS.

BY PAUL D. FOOTE, W. F. MEGGERS AND F. L. MOHLER.

SOMMERFELD has shown there is evidence that enhanced spectra arise from the removal of more than one electron from the normal atom. In the case of the alkali earths the simplest type of enhanced spectra would be produced by the removal of both valence electrons followed by the return of one of these to its equilibrium position in successive interorbital transitions. We have to deal accordingly with a core of net charge $+2e$. The outer structure of the atom is then similar to the simply ionized element of next lower atomic number, with the exception that the net nuclear charge is $2e$ instead of e. Hence the enhanced spectra should be doublets, characteristic of the "arc" spectrum of the alkali metals, but with $4N$ instead of N appearing in the simple spectral series formulæ. Also the separation of the doublets in the enhanced spectra of an alkali earth should be approximately four times that for the corresponding alkali. Sommerfeld has shown that these conditions are well confirmed by existing data—not exactly to be sure, on account of the modifying effects

[1] Fairchild, Jour. Op. Soc. Am., 4, pp. 496–503, Nov., 1920.

arising from the mutual repulsion of the electrons forming the atom, but sufficiently close to indicate that the simple enhanced spectra of the alkali earths arise from the doubly charged atom.

The highest convergence frequency in the arc spectrum (nuclear charge $1e$) of an alkali earth is 1.5S, corresponding by the quantum relation $h\nu = eV$ to the amount of work required to remove one valence electron. Similarly the frequency for the simple enhanced spectrum is 1.5 ⊜ corresponding to the amount of work required to remove the second valence electron after the first has been removed. Some of these latter frequencies have been summarized by Fues.[1]

For magnesium as a particular example we accordingly have the following critical voltages.

This shows that on the basis of Sommerfeld's theory the enhanced spectrum of magnesium should be excited in a low voltage arc by a 15-volt impact—a remarkable conclusion when one considers that these spectra have been in general studied with a capacity discharge at ten to twenty thousand volts. Our experimental work has verified this theory. While only the ordinary "arc" spectrum appears at 8 volts practically every enhanced line listed by Fues is excited in a twenty volt arc. The reason these lines do not appear prominently in an ordinary arc as used by spectroscopists is that electrons of twenty volts velocity are infrequently produced even though the applied potential exceed this. The mean free path is so small that the velocity loss of 7.61 volts, resulting in ionization and the "arc" spectrum, maintains the velocity of most of the electrons at approximately this value or less.

In the present work we have employed a 3-electrode tube with a grid very close to a hot tungsten filament and a plate at a considerable distance, in metallic contact with the grid, the whole being immersed in the vapor at a low pressure. Hence it is possible to develop any desired velocity in so small a distance that collision with atoms is infrequent and the electrons may then pass into the force free space between the grid and plate where collision and emission of radiation take place.

This method of producing enhanced spectra opens a new field whereby the truly enhanced lines may be determined immediately upon examination of the photographic plates, since the enhanced spectrum is not produced below the second ionization point. In ordinary arcs, on the other hand, some electrons are accelerated, without collision, for several mean free paths, far enough to accumulate velocity sufficient for the production of the second type of ionization, with radiation characteristic of the spark. This is probably one reason why so little is known of the enhanced spectrum of mercury. We have photographs for mercury also but have not as yet determined the values of the higher critical voltages.

Bureau of Standards,
Washington, D. C.

[1] Fues, Ann. d. Physik, 63, p. 1, 1920.

ON THE PHOTOELECTRIC EFFECT OF ALKALI VAPORS.

BY JAKOB KUNZ.

ABOUT a year ago E. H. Williams and the writer have shown that cæsium vapor can not be ionized by light of a wave-length shorter than that determined by the equation $hn = eV$, where V is the ionization potential of cæsium vapor. In an article on atomic theory and low voltage arcs in cæsium vapor[1] P. D. Foote has shown that the resonance potential of 1.45 volts gives rise to the single line spectrum, the doublet $1.5s - 2p\frac{1}{2}$. Thus only two inelastic impacts of electrons are known in cæsium vapor, corresponding to 1.45 and 3.88 volts; the former producing the first doublet of the principal series, the latter the entire series. On the basis of Bohr's theory one should expect one line after the other to appear as the potential is increased from 1.45 to 3.88 volts. This successive appearance of lines has indeed been found in mercury vapor by J. Franck.[2] The vapors of mercury and cæsium seem to have different properties, if the results of both physicists are correct.

Low voltage arcs in metallic vapors and in helium have been explained by K. Compton as an absorption of light due to the resonance potential, through neighboring atoms, whereby an electron is lifted from the 1.5S level of energy to the $2p\frac{1}{2}$ level; where it requires a smaller amount of kinetic energy of a colliding electron in order to be removed from the atom. This point of view has been confirmed through the study of the effect of fluorescence on the ionization potential of iodine vapor.[3]

Mr. P. D. Foote draws also the conclusion that electrons revolving not in the $1.5S$ but in the $2p$ ring, should give rise to absorption of the first and second subordinate series, which usually are not absorption lines. We can raise the electron from the stable ring $1.5S$ to the less stable ring $2p$ either by absorption of the corresponding line or by colliding electrons. From the new position the electron may be ejected either by absorption of radiant energy or by incident electrons of proper velocity. If therefore in the photoelectric effect the alkali-vapor is under the influence of the first doublet of the principal series, a smaller frequency of light will be required to ionize the vapor than that of 3.88 volts. So it should be possible to drive an electron from an atom of cæsium vapor by a variety of ways, corresponding to the different absorption lines of the principal series. We may lift the electron from the normal stable orbit to the surface in two or more steps, but always so that $h(v_1 + v_2 + \cdots) = hr = eV$, where $V = 3.88$.

The metal cæsium is sensitive already for infra-red light, while the vapor requires ultra-violet light for the emission of electrons. The vicinity of the atoms in the metallic state seems to loosen the electrons on the surface of the atoms. Hence when the density of the vapor becomes very great, the ionization potentials may become smaller. The ionizing potentials may also ap-

[1] Phil. Mag., Vol. XI., p. 80, 1920.

[2] Zeitschrift für Physik, II., p. 1 , 1920.

[3] PHYS. REV., Vol. XVI., p. 501, 1920.

parently decrease when high electronic currents from very hot filaments are used where a measurable fraction of electrons leave the metal with a high velocity. The experiments on the photoelectric effect of alkali vapors are continued by H. E. Williams at the University of Illinois.

UNIVERSITY OF ILLINOIS.

PHOTO-ELECTRIC POTENTIALS FROM THE RETINA.

BY E. L. CHAFFEE AND W. T. BOVIE.

THE photo-electric currents obtained, when the retina of the eye is illuminated, have been investigated by several experimenters, using as an indicating instrument the capillary electrometer or Einthoven galvanometer. When such current-operated instruments are used, the deflections depend not only upon the electromotive force produced in the eye by photo-electric action, but also upon the resistance of the eye and of the rest of the circuit. In the present work, the study of the photo-electric potentials from the retina has been made by the use of a two-stage vacuum-tube amplifying system connected to operate an Einthoven galvanometer. The use of the amplifiers gives greater sensitivity to the complete measuring system, and what is of greater importance, gives deflections which are proportional only to the potentials produced in the retina instead of depending upon the resistance of the circuit.

Various curves recorded photographically are shown, giving the effects upon the photo-electric potential produced in the retina as the age of the eye after removal increases, as the intensity of the light is varied, and as the time of exposure is changed from very short flashes to exposures of a few seconds. Curves are also presented, giving the photo-electric responses for extremely low ntensities of illumination. The curves are discussed and a tentative theory for the action given.

HARVARD UNIVERSITY,
April 6, 1921.

THE VARIATION OF METALLIC CONDUCTIVITY WITH ELECTROSTATIC CHARGE.

BY HENRY A. PERKINS.

A SIMPLE conception of metallic conduction based upon moving electrons seems to justify the assumption that a negative charge should increase the conductivity of a circuit, and a positive charge should decrease it. This has been tested as follows: A primary coil of about 80 turns was wound upon a glass cylinder inside of which fitted a smaller cylinder similarly wound as a secondary. . The primary was excited from a 60-cycle commercial circuit. The secondary circuit, carefully insulated throughout included a moving coil galvanometer and was given an alternating charge from one terminal of a high tension transformer giving about 5,700 volts, the other terminal being grounded. The two coils wound upon glass acted as a condenser, so that a fairly large charge could be communicated to the insulated system.

In operation the galvanometer was not affected by the charging potential, but the induced alternating current caused a slight vibration of the "spot" on the scale and if it reached too high a value (say 3 milliamperes) produced a small deflection always in the same direction, and due, probably, to constriction of the helical portion of the suspension system. If, however, the induced current and the charging potential were both present, a deflection as high as 30 scale divisions was obtained, which indicated a direct current of about half a microampere. This phenomenon behaved in a perfectly regular and definite manner, and reversed if the phase of the charging E.M.F. were reversed by interchanging the terminals of the transformer. A change of phase less than 180° produced proportional changes in the reading of the instrument.

A simple theory of this effect may be worked out by neglecting the reactance of the circuit, which proved to be very small compared to its resistance. If the charging E.M.F. and induced current are of the same frequency and in the same or opposite phase, the conductance, during one half cycle varies sinusoidally from the normal value to a slightly higher one, and then returns to normal again, while during the other half cycle, it falls to a minimum and returns to normal in the same manner. This causes a pulsating unidirectional current in conjunction with the induced E.M.F. in the secondary circuit. whose value $I = Ey$ when y is the effective change in the conductivity of the circuit. The quantity, y, is proportional to the number, n, of electrons added to or subtracted from the secondary circuit. This number of electrons is equal to EC/e when E is the effective charging potential, C is the capacity of the secondary system and e is the charge of an electron. Similarly the ordinary conductance $Y = 1/R$ is proportional to the effective number of electrons concerned in carrying the current. Calling this quantity Nc, we may assume

$$\frac{Nc}{n} = \frac{Y}{y} = \frac{E}{IR}$$

and substituting for n its value EC/e we obtain

$$Nc = \frac{E\mathcal{E}C}{eIR}.$$

To allow for a change in the phase relation between E and $\mathcal{E}$, this must be multiplied by the cosine of the phase difference ϕ, so that the formula becomes

$$Nc = \frac{E\mathcal{E}C\cos\phi}{eIR}.$$

The values of the variables involved were measured and the resulting calculation gives $Nc = 1.7 \times 10^{13}$. The total mass of copper in the circuit was about 140 grams, so that assuming 29 free electrons (atomic number) for copper and taking Avogadro's constant as 7×10^{23} atoms in 63.6 grams of the wire, we may estimate the total number N of free electrons as 4.4×10^{25} thus giving the ratio $Nc/N = 4 \times 10^{-9}$ which means in round numbers that in this particular circuit only one electron in two hundred million is effective as a carrier of the current.

The size of wire used in the galvanometer is probably No. 36 B. & S. gauge. This would mean a length of 115,000 cm. The current caused by the added electrons, n, is given by $I = nve/l$, where v is their effective velocity and l is the length of the circuit. Calculating n as before from $n = EC/e$, we obtain in the case of a current of 4.4×10^{-7} amperes an effective velocity equal to 8,800 cm. per second in this particular circuit.

TRINITY COLLEGE,
March 31, 1921.

THE DISTRIBUTED CAPACITY OF INDUCTANCE COILS.

BY G. BREIT.

BETWEEN any two portions of an inductance coil there is a certain amount of capacity. This capacity is distributed along the coil and is called the distributed capacity of the coil.

The distributed capacity of the coil gives rise to numerous phenomena, as *e.g.*, non-uniform distribution of current in the coil, a large increase in the resistance of the coil, a change in its apparent inductance, etc.

Some of these phenomena may be explained in terms of a fictitious capacity —called the "effective capacity" of the coil. The definition of effective is given in terms of resonance experiments as follows.

If a condenser of capacity C is connected across the coil terminals the resulting circuit resonates to a certain frequency $\omega/2\pi$. Experiments show that in most cases it is possible to find such numbers L, C_0 that

$$L(C + C_0) = \frac{1}{\omega^2}.$$

The constant C_0, if it exists, is called the "effective capacity" of the coil.

A formula has been derived for the calculation of the effective capacity of a coil. This formula is

$$C_0 = \int_{x_1}^{x^2} \frac{M(x)}{L} \left\{ \int_{1}^{x} \frac{\alpha(x)}{L} dx \right\} dx,$$

where x is an arbitrary parameter along the wire

$$\alpha(x) = \frac{Q(x)}{\left(\frac{di_1}{dt}\right)}$$

where $Q(x)dx$ is the charge between x and $x + dx$, where i_1 is the current at the terminal where L is the self inductance, where $M(x)dx$ is the mutual inductance of the element dx to the rest of the coil.

The formula has been applied to special cases and verified in these cases experimentally. These cases with corresponding formulas are:

(I.) Short single layer solenoid one turn of which has the perimeter l, or pancake of small depth used in elliptical shield of major semi-axis α and minor

semi-axis β and insulated from shield the space between coil and shield having dielectric constant K.

$$C_0 = 0.06952\, Kl \frac{\alpha}{\beta}\ \mu\mu f = \frac{Kl}{16}\frac{\alpha}{\beta}\ \text{c.g.s. electrostatic units.}$$

(II.) Same coil connected to shield at terminal

$$C_1 = 0.06952\, Kl \left[\frac{\alpha}{\beta} + \frac{2}{\log \sqrt{\frac{\alpha+\beta}{\alpha-\beta}}}\right] \mu\mu f$$

$$= \frac{Kl}{16}\left[\frac{\alpha}{\beta} + \frac{2}{\log \sqrt{\frac{\alpha+\beta}{\alpha-\beta}}}\right] \text{c.g.s. electrostatic units.}$$

(III.) Same coil used outside shield insulated

$$C_0 = 0.06952\, Kl\ \mu\mu f = \frac{Kl}{16}\ \text{c.g.s. electrostatic units.}$$

(IV.) Same coil used outside shield and grounded

$$C_0 = 1.1124\left[\frac{1}{16} + \frac{1}{8\log\left(\frac{16R}{a}\right)}\right] Kl\ \mu\mu f$$

$$= \left[\frac{1}{16} + \frac{1}{8\log\frac{16R}{a}}\right] Kl\ \text{c.g.s. electrostatic units.}$$

(V.) Pancake coil of radius a wound to center

(a) when grounded at center

$$C_0 = 0.568\, Ka\ \mu\mu f = \frac{8}{5\pi} Ka\ \text{c.g.s. electrostatic units.}$$

(b) When grounded on the outside

$$C_0 = 0.330\, Ka\ \mu\mu f = \frac{14}{15\pi} Ka\ \text{c.g.s. electrostatic units.}$$

(c) When coil is insulated

$$C_0 = 0.251\, Ka\ \mu\mu f = \frac{32}{45\pi} Ka\ \text{c.g.s. electrostatic units.}$$

(VI.) Multilayer coil of large diameter with circular cross section, a single turn having perimeter l when insulated

$$C_0 = (0.08453\, K_e + 0.08230\, K_o)l\ \mu\mu f$$

where K_e is the effective dielectric constant of the medium inside the coil, and K_o is the dielectric constant of the material outside.

(VII.) Multilayer coil of large diameter with square cross section, a single turn having perimeter l when insulated

$$C_0 = (0.0886\, K_e + 0.1050\, K_o) l\ \mu\mu f.$$

The last two formulas have not been tested experimentally.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

SOME EFFECTS OF THE DISTRIBUTED CAPACITY OF COILS TO GROUND.

BY G. BREIT.

WHEN two condensers connected in series with common terminal grounded are connected across the terminals of an inductance coil as in the figure, their effective capacity in series is not equal to their effective capacity so far as resonance of the coil is concerned. If the capacity of one condenser is C_1 and of the other C_2, the quantity $C_1C_2/(C_1 + C_2)$ is the effective capacity of the two condensers in series. As stated it does not stay constant when C_1 is changed arbitrarily and C_2 is readjusted for resonance.

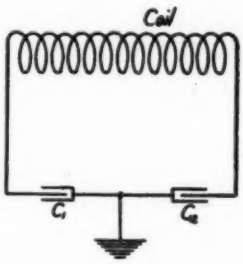

Fig. 1.

A mathematical calculation shows, however, that provided the coil is symmetrical with respect to its two terminals the quantity $C_1C_2/(C_1 + C_2)$ is linearly related to $1/(C_1 + C_2)$.

Experimental verification of this fact has been obtained.

The connections used are shown in Fig. 1 attached.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE HIGH-FREQUENCY RESISTANCE OF INDUCTANCE COILS.

BY G. BREIT.

THE meaning of the term "resistance," so clear for direct current and for low-frequency alternating current, needs careful consideration in the case of high-frequency alternating currents. It is found that in the case of an inductance coil the resistance may assume varying values depending on the particular point in the inductance coil with respect to which the resistance is measured. This is due to the non-uniform distribution of current in the coil.

From a knowledge of the resistance with respect to all points of an inductance coil the current at any place in the coil may be computed for a given distribution of e.m.f.'s in the coil. These results were experimentally verified in the case of coil aerials.

Further, from a knowledge of current distribution and from calculations on skin effect the resistance itself may be calculated. The results of these calculations were tested by experiment.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

METHOD OF MEASURING COIL CAPACITIES AND STANDARDIZING WAVEMETERS.

BY G. BREIT.

A METHOD has been worked out for the adjustment of two frequencies to an accurately known ratio.

The method is based on the fact that an electron tube detector distorts the wave form of the E.M.F. impressed on it and thus produces harmonics of that E.M.F. in its output.

The harmonics produced by a circuit of adjustable frequency are made to give beats with the fundamental of a circuit of fixed frequency. The beats are rectified in an amplifier and are heard as a musical note.

When the frequency of the beats is zero the ratio of the frequencies is exactly a whole number. The whole number may be made very large as, *e.g.*, 100.

There is generally little difficulty in ascertaining the whole number. Thus the method gives frequencies in accurately known ratios.

A knowledge of such ratios is essential to the determination of coil capacities and to the standardization of wavemeters.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

TABLES FOR THE CALCULATION OF THE INDUCTANCE OF CIRCULAR COILS OF RECTANGULAR CROSS SECTION.

BY FREDERICK W. GROVER.

WITH the advance of radio communication, the calculation of inductance has become increasingly important. In the case of the more complicated circuits, the inductance is obtained by first imagining the system to be subdivided into simpler elements for which formulas are already available. By combining the calculated inductances of the elements according to the well known laws for the combination of self and mutual inductances, the solution for the whole circuit is built up.

Perhaps the most important single formulas for the calculation of inductance are, from this standpoint, those for the mutual inductance of parallel, coaxial, circular currents. Formulas have been developed by various investigators which cover, with great accuracy, all the possible cases. The author of the present paper has calculated a table[1] which materially shortens and systematizes the calculation of the mutual inductance of any pair of such circular currents.

Scarcely less basic are the formulas for the inductance of circular coils of rectangular cross section. This is the form of coil usually employed when a larger inductance is required than can readily be obtained from a single layer solenoid. It is true that for higher frequencies, the larger capacity of such a coil is objectionable, but this can be reduced to the order of magnitude of that of a single layer coil by employing a "banked" winding, or a "honeycomb"

[1] Bureau of Standards, Circ. 74, p. 286.

winding, or by the use of insulating material to increase the distance between successive layers of the winding. The inductance of all these forms of coil, and of flat spiral coils as well, may be obtained from the formulas for the inductance of circular coils of rectangular cross section. The relatively small correction to take into account the fact, that the current in an actual coil is not uniformly distributed over the cross section, may be obtained by known formulas for windings of round wire or of strip.[1] It must, however, be noted that the dimensions of the cross section of the equivalent coil of rectangular cross section are not the same as the overall dimensions of the coil. The axial dimension of the cross section of the equivalent coil is to be taken as equal to the product of the number of turns in the layer by the distance between centers of adjacent wires in the layer, while the radial dimension is to be taken as the product of the number of layers by the distance between the centers of corresponding wires in consecutive layers.

In addition to the direct application to multiple layer coils there must be mentioned the possibility of calculating the mutual inductance of coaxial coils of rectangular cross section from a knowledge of the self inductance of such coils. In certain cases the mutual inductance of two coils may be calculated as the difference of the self inductance of certain related coils.[2] This is of especial importance in cases where the coils are nearly in contact, and other methods fail. Yet another application of this basic type of circuit is that of the approximate calculation of the leakage reactance of transformer coils whose dimension along the core is small compared with the dimension of section measured perpendicular to this direction.

Formulas for the inductance of circular coils of small rectangular cross section were long ago developed by Maxwell,[3] Weinstein[4] and Stefan.[5] In recent years Stefan's formula has been extended to apply to coils of larger cross section by Lyle,[6] who prepared tables of coefficients to aid in numerical calculations. Still more recently, the case where the cross sectional dimensions are relatively very large has been covered by the formulas of Butterworth[7] and Dwight,[8] so that it is now possible to treat with accuracy, any coil of this type whatever. Calculations are, however, at best somewhat laborious, and where frequent computations are required, very tedious. Furthermore, without the benefit of considerable experience with these formulas, the computer labors under considerable confusion as to the choice of the formula best suited for the case in hand.

To meet these difficulties, Coursey[9] has prepared curves to simplify the

[1] Bureau of Standards, Sci. Paper 169, p. 140; Circ. 74, p. 261.
[2] Bureau of Standards, Sci. Paper 169, p. 41.
[3] Electr. and Mag., Vol. II, § 706.
[4] Wied. Ann., 21, p. 329, 1884.
[5] Wied. Ann., 22, p. 113, 1884.
[6] Phil. Trans., 213 A, p. 421, 1914. Bureau of Standards, Sci. Paper 320, p. 569, 1918.
[7] Phil. Mag., 29, p. 578, 1915.
[8] Proc. A.I.E.E., 1919.
[9] Proc. Phys. Soc. London, 31, p. 155, 1919.

calculations. As reproduced by the printer, values of the inductance may be determined by graphical interpolation from these curves to one per cent. or perhaps a little better. In computing the curves the method of geometric mean distance was used. This is inadequate to give results with the accuracy just stated in every case when the thickness of the winding is greater than about three tenths of the mean diameter of the coil, the error being greater the thicker and longer the coil, that is, the greater the relative dimensions of the cross section. In cases of coils with very thick windings the error may easily reach ten to twenty per cent. In addition, Coursey's curves do not cover the case of coils whose radial dimension exceeds the axial (pancake coils).

To make rapid calculations possible with an accuracy better than one tenth of one per cent., the author of the present paper has calculated for the Bureau of Standards, constants which cover the whole range of possible coils. On account of the difficulties in the reproduction of curves to give the accuracy desired, it has seemed advisable to present the results in the form of tables. This does not, of course, prevent the use of curves drawn, as desired, from the data of the tables.

The calculation of the inductance using these tables is based on the use of two formulas which may be written as follows:

$$L = 0.002\pi^2\left(\frac{2a}{b}\right)n^2aK' = 0.002\pi^2\left(\frac{2a}{b}\right)n^2a(K - dk)\text{microhenrys}, \quad \text{(A)}$$

$$L = 0.001n^2aP' = 0.001n^2aPf \text{ microhenrys}, \quad \text{(B)}$$

where L = inductance with uniform distribution of the current,
a = the mean radius of the coil,
b = the axial dimension of the cross section,
c = the radial dimension of the cross section,
n = the total number of turns.

The quantities K', P', dk and f are functions of the two parameters

$$c/2a = \frac{\text{thickness}}{\text{mean diameter}}$$

and b/c or c/b, the ratio of the two dimensions of the cross section. The values in the tables are given for values of b/c or c/b in steps of 0.1, and for $c/2a$ in steps of 0.1 with the added values of 0.025 and 0.05.

Formula (A) is especially useful for relatively long coils, while formula (B) applies best to pancake coils, although both may be used in any case, and thus provide the means of checking the calculations.

When the given case may be treated without interpolation in the tables, the quantities K' and P' should be used. Otherwise, greater accuracy is attained by interpolating dk or f, and using the alternative forms of the formulas. The quantity K is the value of K' for the single layer coil of the same length and mean diameter as the actual coil. A table of this quantity was published by

Nagaoka.[1] The factor P is the value of P' applicable to a disk coil of the same mean diameter and radial dimension c as the coil in question. A table of values of P for values of $c/2a$ in steps of 0.01 has been prepared by the author using the formulas of Rayleigh and Niven, Lyle and Spielrein.[2] Interpolation of P may be very accurately and easily accomplished from this table.

All the values in the tables have been calculated from the various formulas mentioned above, that formula being chosen in each case whose convergence is most satisfactory. In certain cases where two overlapping formulas could be employed, both have been used as a check on the result.

ELECTRICAL ENGINEERING DEPT.,
UNION UNIVERSITY,
April 4, 1921.

ON SOME NEW FORMULÆ FOR THE DIRECT NUMERICAL CALCULATION OF THE COEFFICIENT OF MUTUAL INDUCTION OF COAXIAL CIRCLES.

BY LOUIS V. KING.

IF we start with the positive numbers (a_0, b_0), the arithmetical operations given below are easily carried out on a modern calculating machine.

$$\begin{array}{lll} a_0 & b_0 & \\ a_1 = \tfrac{1}{2}(a_0 + b_0) & b_1 = \sqrt{a_0 b_0} & c_1 = \tfrac{1}{2}(a_0 - b_0) \\ a_2 = \tfrac{1}{2}(a_1 + b_1) & b_2 = \sqrt{a_1 b_1} & c_2 = \tfrac{1}{2}(a_1 - b_1) \\ \cdots & \cdots & \cdots \\ a_n = \tfrac{1}{2}(a_{n-1} + b_{n-1}) & b_n = \sqrt{a_{n-1} b_{n-1}} & c_n = \tfrac{1}{2}(a_{n-1} - b_{n-1}) \\ \cdots & \cdots & \cdots \end{array} \qquad (1)$$

The a's and b's converge with great rapidity to the same limit, which may be denoted by a_n.

The array of numbers given above originated with Lagrange (1784–85) and was employed by Legendre (1786) in computing his well-known tables of elliptic integrals. The use of the above process for the numerical evaluation of elliptic integrals was discussed independently by Gauss (1818), and was called by him the "algorithm of the arithmetico-geometrical mean."

Denoting by K and E the complete elliptic integrals of the first and second kinds, and by k and k' the modulus and its complement, it may be proved that

$$K = \tfrac{1}{2}\pi/a_n \quad \text{and} \quad (K - E)/K = \tfrac{1}{2}(c_0^2 + 2c_1^2 + 4c_2^2 + \cdots + 2^n c_n^2 + \cdots) \qquad (2)$$

both well-known results.

It does not appear to have been generally remarked hitherto that $2^{-n} \log (a_n/c_n)$ converges to the limit $\tfrac{1}{2}\pi K'/K$. The writer has been unable to find this result explicitly stated by Gauss or by any subsequent writer, although the following formula

$$\tfrac{1}{2}\pi K'/K = \tfrac{1}{2} \log (4a_1/c_1) - \sum_1^\infty (\tfrac{1}{2})^n \log (a_n/a_{n+1}) \qquad (3)$$

[1] Jour. Coll. Sci. Tokyo, 27, art. 6, p. 18, 1909.

[2] Arch. f. Elekt., 3, p. 187, 1915.

derived from it was found among a number of formulæ in Gauss' posthumous papers, stated without proof. An equivalent result is given however, by Legendre.

The use of these *arithmetico-geometrical scales* for the direct numerical computation of formulæ involving elliptic integrals is now made possible by the modern calculating machine on which square-root operations are easily carried out.

Let A and a be the radii of two co-axial circles and d the distance between their planes. If we compute $r_1^2 = (A + a)^2 + d^2$ and $r_2^2 = (A - a)^2 + d^2$, and the arithmetico-geometrical scale ($a_0 = r_1$, $b_0 = r_2$) is then derived, we find that Maxwell's well-known formulæ for the mutual inductance of the two circles may be written

$$M = (2\pi^2/a_n)(c_1^2 + 2c_2^2 + 4c_2^2 + \cdots + 2^{n-1}C_n^2 + \cdots). \qquad (4)$$

This formula is extremely convergent and convenient to use in *all circumstances*, three terms being usually sufficient to give seven-figure accuracy.

When the coils are very close together, ($r_2/r_1 > 0, 1$) there is some advantage in computing the complementary scale, ($a_0' = r_1 + r_2$, $b_0' = r_1 - r_2$), which is then extremely convergent, a_2' and b_2' being the same to seven figures. We then find that

$$M = 2\pi a_n' \left[\frac{(r_1^2 - 2c_2'^2 - 4c_3'^2 - \cdots)}{a_n'^2} \left(\log \frac{4a_3'}{r_2} - \tfrac{1}{2} \log \frac{a_2'}{a_3'} - \tfrac{1}{4} \log \frac{a_3'}{a_4'} - \cdots \right) - 2 \right] \qquad (5)$$

The new formulæ (4) and (5) when applied to numerical examples show a decided advantage over existing methods of computing M.

Formulæ of this type, making use of trigonometrical recurrence-formulæ combined with the A.G.M. scales have been derived for the direct numerical calculation of the self and mutual inductances of single layer coils.

McGILL UNIVERSITY,
MONTREAL, April 6, 1921.

NOTE ON THE EARTH'S MAGNETIC FIELD.

BY W. F. G. SWANN.

H. LAMB has shown that if the conductivity of the material of the interior of the earth were as great as that of copper, a system of currents such as would account for the earth's magnetic field would if once started within the earth, take ten million years to die to $1/e$ of their original values even though no external electromotive forces were present to maintain them. This result is due to the very great influence of self induction in the case of a body of the size of the earth.

In the present note it is shown that if the earth had been permanently magnetized at some time, and if the permanent magnetism were suddenly destroyed, the induced currents set up would perpetuate the original magnetic

field to such an extent that, for a conductivity equal to that of copper, several million years would elapse before the field had diminished to $1/e$ of the value corresponding to the original magnetization.

A further extension of this idea shows that if the earth's magnetism be pictured as due to the superposition of two uniform magnetizations, one parallel to the geographic axis and one perpendicular thereto, and if the secular rotation of the pole be pictured as a uniform rotation of the second named magnetization relative to the earth with a period corresponding to one complete rotation in 500 years, then we are unable to admit more than a certain limiting conductivity for the earth's interior, as otherwise the induced currents set up by the rotation would be too large to be consistent with the rotation postulated. The limiting conductivity in question is of an order of magnitude certainly less than 1/5,000 that of copper.

UNIVERSITY OF MINNESOTA.

THE ELECTRICAL RESISTANCE OF THE HUMAN BODY.

BY F. WENNER, JAS. S. MARTIN, NYNA FORMAN.

FROM a study of the results that have been published on the measurements of the electrical resistance of the human body, we find that the values obtained differ widely depending upon the methods of measurements and the conditions under which the measurements are made.

About two years ago, Professor Gildemeister[1] made an extensive investigation of the resistance of the human body. In summing up the results of his measurements, we find that he obtained values for the electrical resistance of substantially the same parts of the body varying from 148,000 ohms to 68 ohms, depending upon the duration of current flow, the voltage, and the frequency. On increasing the frequency by a factor of 430 he found the resistance to decrease by a factor of 15. He discusses his data fully and reaches the conclusion that in the case of accident, when the skin is burned, the resistance falls to a very low value.

Dowse and Iredell[2] obtained much more consistent data and it is interesting to note that while they changed the frequency by a factor of 1,000 they found the impedance from hand to hand to change by a factor less than two. They further conclude that the change in resistance is less than the change in impedance. The results cannot therefore be considered to be in accord with those obtained by Professor Gildemeister.

The area of the electrodes, condition of the skin where the test current enters and leaves the body, and electrolytic and electrostatic actions taking place at or in the vicinity of the electrodes is a factor which apparently affected all the values previously obtained. We, therefore, thought that it might be of interest to take a few measurements in such a way that this factor could have no effect upon the values obtained.

[1] Elektrotechnische Zeitschrift, 1919, Vol. 40, p. 463.

[2] Archives of Radiology and Electrotherapy, 1920, Vol. 25, p. 33.

An essential feature of the plan adopted for attacking the problem is the use of four connections to the body; two of these serve to lead the test current to and from the body and the other two serve to bring the potential difference, developed in a certain part of the body by the test current, to a place where it can be measured. This is a method used for the purpose of eliminating the effects of contact resistances where we wish to determine the resistance of short and thick metallic conductors. Other features of the plan are the use of an alternating test current, a method of measurement which requires the passage of little or no current through the potential terminals, and a detector which responds to alternating current only. It was thought that possibly this plan would give results sufficiently consistent to show definite differences between individuals or changes in the same individual.

During July, 1920, a series of measurements was made using the left hand and foot as current terminals and the right hand and foot as potential terminals. The arrangement was such that the magnitude of the test current could readily be changed by a factor of two, and its frequency changed from 25 to 100 cycles.

No change in resistance was observed on changing either the frequency or the voltage, though the sensitivity was such that changes of a few per cent. could have been detected. Consequently in a second series of measurements made in November, no provision was made for changing the frequency of the test current and to avoid the operation of a special machine 60 cycles was used.

The resistance of the trunk was found to vary with the position of the individual or to be larger with the muscles tense, than relaxed, and measurements on the same individuals made at different times give slightly different values for the resistance of the trunk of the body. For most individuals the resistance of the trunk was found to be between 25 and 30 ohms.

These we believe to be the first measurements giving any definite idea as to the resistance of the "vital" parts of the body.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

NEW METHODS FOR MAINTAINING CONSTANT FREQUENCY IN HIGH-FREQUENCY CIRCUITS.

BY W. G. CADY.

As stated in a former communication,[1] a plate properly cut from a piezo-electric crystal and provided with metallic coatings vibrates mechanically when the coatings are connected to an E.M.F. of high frequency, provided that the frequency is sufficiently near the natural frequency of mechanical vibration. The absorption of energy gives the plate an effective series resistance which is a maximum at the resonant frequency. The effective parallel capacity of the plate is a maximum at a frequency slightly below resonance, and a minimum at a frequency slightly above resonance, passing through the normal value at resonance. The total range in capacity is greater, the less

[1] PHYS. REV., April, 1921.

the vibrations are damped, and in the case of quartz it may be more than twenty times as great as the normal capacity of the plate. The accompanying change in frequency may be a very small fraction of a per cent.

This dependence of capacity upon frequency may be utilized as a frequency stabilizer, by simply connecting the piezo-electric plate in parallel with the tuning condenser of an ordinary vacuum tube oscillating circuit. When the frequency is slightly above that at which the plate is in mechanical resonance, an increase in the capacity of the variable condenser is accompanied by a nearly equal decrease in the effective capacity of the plate, so that the frequency remains constant within exceedingly narrow limits. If the condenser reading is increased beyond an amount corresponding to the maximum plate capacity, the plate suddenly stops vibrating, and the frequency abruptly decreases by a large amount. A similar process, in the reverse sense, takes place on decreasing the capacity of the variable condenser.

For example, a certain quartz plate 3.9 cm. long vibrated at a frequency of about 69,700. A change in the variable condenser which, when the quartz plate was removed, altered the frequency by 3 per cent., varied it by less than one part in 20,000 when the quartz was in circuit. This plate had coatings so small that its normal capacity was only about 0.67 micro-micro-farads; yet when vibrating, its effective capacity varied from 10 to -9 M.M.F.

A similar stabilizing effect can be obtained for any frequency over a very wide range by selecting a plate of the right length.

Two other methods of connecting a quartz plate may be mentioned, which under favorable conditions have been found to exert an even more marked stabilizing influence upon the frequency. In one of these, the plate is provided with two pairs of coatings, one pair being connected in parallel with the tickler coil in the anode circuit, the other in parallel with the grid condenser and leak. If the circuit without the plate is on the verge of oscillating, then when the plate is introduced, the vibrations of the quartz furnish the "feed-back," and the circuit oscillates at a frequency determined solely by the plate itself. By a proper choice of circuit constants, the plate can be made to vibrate at either the fundamental frequency or the second or third harmonic.

In the third method, the plate has but one pair of coatings, connected in parallel with the grid condenser and leak. Here again, by a suitable choice of circuit constants, the electrical frequency will be found to be determined by the vibrations of the plate. In this, as in the other methods, the frequency is practically unaffected by small changes in capacity or self-inductance of the circuit.

WESLEYAN UNIVERSITY,
MIDDLETOWN, CONN.

MAGNETIC ROTATORY DISPERSION IN TRANSPARENT LIQUIDS.

BY R. A. CASTLEMAN, JR., AND E. O. HULBURT.

Theoretical.—From the electron theory of H. A. Lorentz a theoretical formula was derived which expressed the relation between the angle of magnetic rotation

of the plane of polarization of plane polarized light produced by the magnetized medium, the wave-length of the light, the strength of the magnetic field, and other physical quantities. Absorption of light in the medium was neglected in the theory.

Experimental.—The magnetic rotation angle of a number of transparent liquids (carbon disulphide, benzene, nitrobenzene, etc.) for a series of wave-lengths in the visible spectrum was measured. The apparatus consisted of a grating monochromatic illuminator, two nicol prisms, and a Ruhmkorff magnet between whose poles was placed a cell containing the liquid under examination.

Comparison of Theory and Experiment.—The magnetic rotation angles for various wave-lengths were calculated from the theoretical formula and compared with the observed ones. It was found that the theory agreed with the observations for the longer wave-lengths but that for the shorter ones the theoretical rotations were greater than the observed, the discrepancy increasing with decrease in wave-length. It is pointed out that if the effect of absorption were incorporated in the theory the above discrepancy would be lessened. Lack of experimental data precluded consideration of absorption.

Value of e/m.—The values of the ratio of the charge to the mass of the dispersion electrons were calculated from the observations by means of the theoretical formula, and they were found to lie between 0.49×10^7 and 1.12×10^7, when the electronic charge is expressed in c.g.s. electro-magnetic units.

JOHNS HOPKINS UNIVERSITY.

TOTAL EMISSIVE POWERS AND RESISTIVITIES OF TUNGSTEN AT INCANDESCENCE.

BY A. G. WORTHING AND W. E. FORSYTHE.

TOTAL emissive power and resistivity relations of metals at incandescent temperatures are readily obtained from current, voltage and temperature measurements on filaments mounted in well evacuated lamp bulbs. The process is considerably simplified when the cooling effects of leads are eliminated by means of fine potential wires of the same material attached to the filaments at considerable distances from the cooling leads.

In this work, lamp A–33, containing a tungsten filament, 0.257 mm. in diameter and about 30 cm. in length with potential leads so attached as to permit of voltage measurements across the central third (10.85 cm.) of its length, has been taken as a standard with which others have been compared. Certain data on this lamp have been discussed elsewhere.[1] Due to an accidental error in platting, the possibilities of the data were not appreciated at that time. Since then, however, the conclusions to be drawn from data on this lamp for the same temperature range, as well as predictions for greatly increased temperature ranges, have been verified by many tests on other

[1] Worthing, PHYS. REV., 10, p. 377, 1917.

lamps. Moreover, so far as individual variations between lamps are concerned, A–33 seems to represent a fair average.

To well within the accuracy of the potentiometer and temperature measurements, the resistance-temperature and emission-temperature relations for this lamp are given by

(I.) $$\frac{R}{R_0} = \left(\frac{T}{T_0}\right)^{1.200},$$

and

(II.) $$\log E = 3.680\,(\log T - 3.3) - \frac{1040}{T} + 1.900,$$

where E is expressed in watts/cm² and T in ° K. Deviations from fulfillment for the latter equation are not greater than those due to temperature errors of 0.5° K.

How well the resistance-temperature relation is fulfilled is shown in the accompanying plat in which data on other lamps are included for temperatures ranging from 1400° to 2700° K. That the relation in each case is given by equation I. is evident. It is equally evident that the exponent changes as we pass from one lamp filament to another. This probably depends upon whether the filaments are thoriated or not, and also certainly upon the history of the filament in the lamp. On this account, the actual exponents corresponding to T–110 less T–111 and T–104 less T–101 may not be as significant as in the other instances. Tests show some deviation from equation I. as room temperatures are approached. On the other hand, tests carried up to 3200° K. indicate that the equation holds perfectly for increasing temperatures. Absolute resistivities may be obtained by taking 59.2×10^{-6} ohm cm. as the absolute value at 2000° K.

How well equation (II.) holds is shown in Fig. 4 of the paper referred to above. Data on various other lamps which were not broken up for filament dimensions show agreement with the curve presented there so far as variation with temperature is concerned. A recent attempt to extend that curve to noticeably higher temperatures has yielded results in accord with it and with equation (II.) up to 3200° K.

Combining equation (II.) with the Stefan-Boltzmann constant yields the total emissive power of tungsten as a function of temperature. Results based on 5.72×10^{-12} watts/cm² deg⁴ for that constant and the temperature scale[1] for tungsten adopted by the General Electric Company are given in the table.

Starting with low values at low temperature, there is seen to be a gradual increase of the exponential type as though there were a limiting value in the neighborhood of 0.40, as might be expected from a knowledge of emissive powers in the visible and the ultra-violet.

An attempt to compute the total emissive powers from the resistivities at low temperatures as was done by Foote[2] for platinum using the equation

$$e_t = 0.5736\sqrt{\rho T} - 0.1769\rho T$$

[1] Gen. Elec. Rev., 20, p. 819, 1917.
[2] Bull. Bur. Stds., 11, p. 607, 1915.

yielded values considerably lower than the true values, indicating, as was found by Davison and Weeks[1] for platinum, that there were considerable deviations from the cosine law of emission for the total radiation of tungsten. Similar computations, using reflectivities of tungsten at room temperature as determined by Coblentz[2] and the variation with temperature as determined by Weniger and Pfund,[3] show a similar lack of fulfillment of the cosine law. Using Weniger and Pfund's data at room temperature (unpublished) in place of Coblentz' showed still greater variations. At 2540° K. the operating temperature of the ordinary vacuum tungsten lamp, the two values thus computed were found too low by $7\frac{1}{2}$ per cent. and 13 per cent. respectively. It is hoped that a test to show the cosine law variation for the total radiation may be carried out soon at our laboratory.

Total Emissive Powers of Tungsten.

T in °K.	E (tungsten) in $\frac{\text{watts}}{\text{cm}^2}$	E (black body) in $\frac{\text{watts}}{\text{cm}^2}$	$\frac{e}{t}$
1000	0.57	5.72	0.100
1500	5.64	29.0	.194
2000	24.2	91.5	.264
2500	69.8	223.	.313
3000	161.	463.	.347
3500	317.	858.	.369

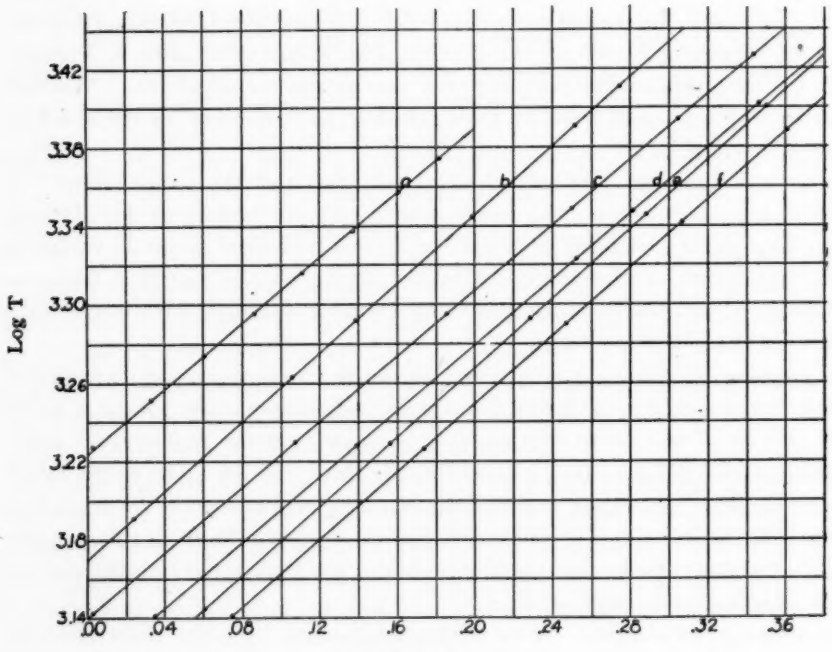

Fig. 1.

[1] PHYS. REV., 17, p. 261, 1921.

[2] Bull. Bur. Stds., 14, p. 312, 1918.

[3] PHYS. REV., 14, p. 427, 1919.

Temperature Resistance Relations for Various Tungsten Lamps.

Curve	Lamp.	Exponent in Equation (I.).
a	T–1	1.210
b	T–98	1.135
c	A–33	1.200
d	A–30	1.187
e	T–110 less T–111	1.128
f	T–104 less T–101	1.160

NELA RESEARCH LABORATORIES,
NELA PARK, CLEVELAND, OHIO,
April 7, 1921.

COLOR TEMPERATURE OF HIGH EFFICIENCY LAMPS.

BY W. E. FORSYTHE.

THE efficiency and life of incandescent lamps depend upon the temperatures at which they are operated, the efficiency being greater for high temperatures and the life shorter. Lamps are therefore operated at temperatures which will give the desired life. The integral color of the light given by the lamp also depends upon the temperature, the light being bluer for the higher temperatures.

It has been found by experiment that the light from incandescent lamps can be matched in color with that from a black body at some particular temperature which temperature is called the color temperature. Making use of this fact the quality of the light given by the different lamps can be given by its color temperature.

Several different kinds and types of incandescent lamps operated at certain efficiencies have been color-matched with a black body and the results are given in Table I.

Lamp.	W. P. S. C.	Temperatures Degrees K.	Color Temperature.
50-watt Carbon	3.76	2115[1]	2156
50-watt Gem	3.1	2180[1]	2195
50-watt Tantalum	2.56	2160[2]	2260
40-watt Mazda B	1.36	2410	2460
100-watt Mazda C	1.00	2745	2740
500-watt Mazda C	0.72	2835	2880
1000-watt Mazda C	0.62	3010	2985
1000-watt Stereopticon	0.52	3185	3175
900-watt Movie	0.46	3290	3220

[1] True temperature calculated from brightness temperature, assuming an emissive power of 80 per cent.

[2] True temperature calculated from brightness temperature, assuming an emissive power of 48 per cent.

These temperatures are based upon the assumption of Wien's equation with c_2 taken as 14350 μ deg. and upon the melting point of gold taken as 1336° K. On this scale the melting point of palladium has been found to be 1828° K. For convenience in the calibration of optical pyrometers, a black body held at the melting point of palladium is used as the point of reference.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
April, 1921.

THE DETECTING EFFICIENCY OF THE RESISTANCE-CAPACITY COUPLED ELECTRON TUBE AMPLIFIER.

BY E. O. HULBURT.

Definition of Detecting Efficiency.—The detecting efficiency is defined by the ratio b_0/A^2, where b_0 is the rectified high frequency component of the output plate circuit produced by a high frequency voltage of magnitude A impressed on the grid of the input tube.

Derivation of Theoretical Formula.—A theoretical formula is derived for the detecting efficiency of an amplifier consisting of two tubes coupled by resistances and capacities.

Apparatus.—Measurements of the detecting efficiency were carried out by means of a *condenser potential divider*, and a sensitive *galvanometer* connected in the plate circuit of the detector tube.

Comparison of Theory with Experiment.—The detecting efficiency of the two tube amplifier for various frequencies and coupling capacities was measured and found to agree with the theoretical formula with a fair accuracy. The detecting efficiency increased with the wave-length throughout the range from 400 to 1,600 meters. The *current* and *sound intensity* amplification were determined.

Comparison with Other Amplifiers.—Measurements of the detecting efficiency for various wave-lengths of a two and three tube resistance-capacity coupled amplifier, of a three-tube transformer coupled amplifier, and of two two-tube transformer coupled amplifiers were obtained which gave the relative merits of the various amplifiers as far as their detecting efficiency for an unmodulated signal was concerned.

JOHNS HOPKINS UNIVERSITY.

RADIO SIGNAL FADING PHENOMENA.

BY J. H. DELLINGER AND L. E. WHITTEMORE.

A STUDY has been made of the phenomena exhibited by variations of intensity of received radio signals. The following four functions characteristic of radio transmission were included in the study: intensity of signals, fading of signals, direction of waves, strays. Particular attention was given to the results of some recent tests of short wave radio fading conducted with the coöperation

of the American Radio Relay League. It is concluded that the sources or causes of fading or of strays are in the atmosphere between the earth's surface and the Heaviside surface which is a conducting surface some sixty miles above the earth. Daytime transmission is largely carried on by means of waves moving along the ground, while night transmission, especially for great distances and short waves, is by means of waves transmitted along the Heaviside surface. Waves at night are thus free from the absorption encountered in the daytime but are subject to great variations caused by irregularities of the ionized air at or near the Heaviside surface. These variations probably account for fading.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

SOME PHYSICAL PROBLEMS OF AIRCRAFT RADIO.

BY L. E. WHITTEMORE.

AMONG the lines in which the physicist can be of service to industry and commerce is the solution of problems of radio communication, of which an important example is the development of the use of radio for communication with aircraft.

A statement is given of several problems involved in the reception of radio signals on aircraft and on the ground from aircraft and in the transmission of signals from aircraft and from the ground to aircraft. The progress made to date on several of these problems is summarized.

It is to be noted that these questions are of interest to both the mathematical and the experimental physicist. Their solution will not only make aerial navigation safer and transportation by aircraft more useful, but will also aid greatly in developing the methods of radio communication for other purposes.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE DESIGN OF A STANDARD WAVEMETER.

BY R. T. COX.

IT is explained that the increasing demand for more precise wave-length measurement is making the design of wavemeters more and more a problem for the physicist rather than the engineer. The mechanical and electrical requirements which the necessity of precise measurement imposes on each part of a wavemeter are stated. The way in which these requirements have been met in the latest wavemeter of the Bureau of Standards is described. Possible modifications of the design in the interest of economy, where the requirements are not so exacting, are indicated.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE HIGH-FREQUENCY IMPEDANCE OF RADIO TELEPHONE RECEIVERS.

BY C. T. ZAHN.

IN order to study the behavior of certain radio circuits where telephones are used, it is necessary to know the radio-frequency impedance, as well as the audio-frequency impedance and the acoustical properties. On account of the capacities between the closely spaced leads used with the receivers the impedance is very different from that of the receivers alone.

It was the purpose of the work here described to measure the impedance at various radio-frequencies of different types of head-set in use. The measurements were made by measuring the resistance of a simple tuned circuit by the resistance-variation method, and then connecting the telephones across the condenser and returning and measuring the resistance again, and computing the impedance from these measurements.

The head-sets have a critical frequency where the reactance changes sign and near which the resistance is maximum. This critical frequency for the receivers tested varies in the approximate range of 8,000–50,000 cycles per second, the maximum values of resistance and reactance being of the order of 100,000 ohms.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A RECORDER OF RADIO TIME SIGNALS.

BY E. A. ECKHARDT AND J. C. KARCHER.

A RADIO time signal recorder was developed by the authors which permits simultaneous recording of radio time signals or other radio signals and local signals such as the breaks of a chronometer, or signals made by the pressing of a key by an observer. The recording of all signals is accomplished by a single chronographic pen and the circuits are so arranged that the lags are due to differences in the raction times of the relays involved instead of the reaction times themselves. The apparatus provides a means for obtaining simultaneous time indices in connection with recording operations going on simultaneously at different points without the necessity of wire connections between them. Various velocity determinations are thus made possible, as, for example, of earth waves, sound, etc.

Radio signals from Lyons, France (3,800 miles air line) have been recorded with regularity. In such recording the receiving-recording apparatus contained only one electron tube of the ordinary receiving type in most cases, and never more than two. The antenna used was a 50-foot 6-wire flat-top antenna, 8 feet above the roof of a building 57 feet high.

The receiving equipment involves a regenerative electron tube circuit the plate circuit of which contains the windings of a telegraph relay. The grid potential of the tube is adjustable by means of a potentiometer. Considering the regenerative circuit alone there is a critical grid potential at which the

circuit will become self-oscillatory as this potential is approached from more negative values. The passing of the circuit from the non-oscillating to the oscillating state is accompanied by a large rise in the mean plate current. In the circuits we are using this rise is from a small fraction of a milliampere to several milliamperes. The relay in the plate circuit is so adjusted that it will be released against an outer contact at the smaller value and pulled against an inner contact at the larger one. The opening of the outer contact results in an excursion of the recording pen from the datum line. The closing of the inner contact short circuits a low resistance coil of a few turns placed between the regenerating coils and thus stops the oscillations. The opening of this contact does not involve any readjustments in the regenerative circuit and consequently permits working very close to the critical potential. In this manner a greater sensitivity has been obtained than any now on record.

If the regenerative circuit be suitably related to a radio receiving antenna the rise in potential due to radio signals may be used to raise the grid potential above the critical value. The operation is then as described above.

In order to record local signals as well as the radio signals the pen controlling magnet has a double winding. The closing of the local circuit then magnetizes or demagnetizes the pen controlling magnet according as the other circuit is open or closed. In this way the two kinds of records may be obtained simultaneously without mutual interference.

Bureau of Standards,
Washington, D. C.,
April 6, 1921.

An International Series of Radio Audibility and Direction Measurements.

By Carl Kinsley.

In August, 1919, a series of measurements were made simultaneously in England, France, Italy and the United States. In England two stations were used, in France there were also two and in the United States four stations were employed which were widely separated in Maine, Arizona, Texas and the District of Columbia. These experiments consisted of audibility and direction measurements upon the signals sent out by the Annapolis arc transmitter.

A comparison of the observations has been made and it is found that the variations of audibility at the different stations does not occur simultaneously. During the course of the experiments the "fading" of the signals was particularly marked at times other than the twilight hours. The most pronounced variations of direction occurred after midnight. These did not take place simultaneously at the different stations. These results confirm previous experiments which were made at stations less widely separated.

There seems to be no reason for attributing the variations to the transmitting station. The variations in audibility and in direction do not in general occur at the same time and the largest change of either is noticeably accompanied

by stability in the other. It is probable that the causes may be different—possibly absorption alone is responsible for the audibility variations. A variation in direction of thirty degrees was noted in these experiments and it seems probable that a reflection of the electromagnetic wave at a point not distant from the observer has produced a distorted resultant wave at the receiving station.

DETECTION COEFFICIENT OF THERMIONIC VALVES.

BY E. L. CHAFFEE.

THE detection coefficient of a thermionic valve, when used without a stoppage condenser, is ordinarily derived considering only the curvature of the plate-current grid-voltage curve. Such methods neglect the current taken by the grid, the drop in voltage in the grid circuit as a result of this grid current, and the consequent rectified plate-current. In this paper, an expression is derived theoretically for the detection coefficient, taking into account the grid circuit effects, as well as the plate circuit rectification. The expression is given below.

$$D = \frac{\text{Rectified Plate Current}}{(\text{Applied Grid Voltage})^2} = A \frac{K_c^2}{(K_c + K_g)^2} \left\{ \frac{\partial \sigma}{\partial E_g} - \frac{\mu K_p}{K_c + K_g} \left(\frac{\partial K_g}{\partial E_g} \right) \right\}.$$

Where A is a constant, K_c is the equivalent conductivity of the grid circuit exterior to the tubes,

$$K_g = \frac{\partial I_g}{\partial E_g}$$

is the conductivity of the grid to filament space,

$$K_p = \frac{\partial I_p}{\partial E_p}$$

is the conductance of the plate to filament space with grid voltage constant, μ is the ordinary amplification factor, and

$$\sigma = \frac{\partial I_p}{\partial E_g}$$

is the mutual conductance.

It is shown that the second term, which is due to the grid current, may be in actual practice a very large factor in determining the sensitivity of a tube when used as a detector. Experimental curves are given both on hard tubes and tubes containing a trace of gas, and the experimental results compared with the theoretical expression.

CRUFT LABORATORY,
HARVARD UNIVERSITY,
April 6, 1921.

THE RADIO RESEARCH FIELD.

BY J. H. DELLINGER AND L. E. WHITTEMORE.

RADIO communication is a rapidly growing subject both technically and commercially. It was given a great stimulus by the war and is now becoming

widely used for government, commercial and private communication. Its place will undoubtedly be that of supplementing the wire telegraph and telephone service rather than supplanting it, the linkage between the two methods of communication being shown by recent work on wire-radio systems and by several demonstrations where part of the distance was covered by wire lines and part by radio. The broadcasting by radio telephony of information which has been collected by wire is becoming more common and offers almost unlimited possibilities.

Many schools and universities have installed courses in radio communication, and research work is being done by them as well as by many private experimenters and the large commercial companies which are engaged in operating radio stations and selling radio apparatus.

Much has been learned about the principles of radio communication and the behavior of radio circuits since the use of the electron tube has made precision measurements possible. The Radio Laboratory of the Bureau of Standards has been attempting to serve in this connection by the development of methods of measurement and the study of fundamental principles as well as by encouraging the use of radio communication and radio methods for many purposes.

During the course of work on this subject suggestions continually arise for further research and it is obviously impossible for a limited staff to conduct work on all of them. The Bureau has coöperated with other investigators in the past by suggesting suitable research subjects, and has prepared a classified list of radio research subjects with the hope that it may be suggestive to many who are anxious to contribute to the development of this field of knowledge. Radio methods offer a convenient avenue of approach to many problems in electric waves outside the strictly radio field. The instruments and methods of radio research are so diversified as to offer a broad training to the research worker.

The list of research problems has been classified in accordance with an extension of the Dewey decimal classification applied to radio, which has been developed by the Bureau, in order that related subjects may be conveniently grouped near one another in the list. The classification is useful in classifying other radio material such as reference abstracts, drawings, books, reports, correspondence, etc.

The classification by itself serves to indicate the wide scope of the radio research field, and the detailed statements under the several subjects emphasize the large number which await solution. The outline given is of course not complete nor is it uniformly detailed. But it is hoped that it will serve to show the opportunity and need for a great deal of work in the radio research field. Fortunately, many of the experimental researches may be conducted with relatively cheap and simple apparatus.

Bureau of Standards,
Washington, D. C.

ON EMERGENT β RAYS FROM MATERIALS OF DIFFERENT ATOMIC WEIGHT AND DIFFERENT THICKNESS PRODUCED BY γ RAYS: STUDIED BY THE STATISTICAL METHOD.

BY ALOIS F. KOVARIK.

IN the study of the number of γ rays from a gram of radium per second it seemed necessary to perform some experiments on the emergent β rays produced by γ rays in passing through matter, inasmuch as the method of observation was the statistical method of counting individual β particles or groups of β particles simultaneously produced, while the published results all deal with the ionization method. In connection with the above problem two things are of importance, namely, the variation of the emergent β radiation with the thickness of material used and the variation with the material. Eve has given some results on both of these problems using the ionization method in his observation. Eve and Bragg and Madsen obtained curves in general agreement with the equation obtained on the supposition that the emergent rays are due to absorbed γ ray energy and that both radiations are absorbed in passing through matter. Eve and Hackett have shown that the emergent radiation does not increase regularly with atomic weight, but decreases from Al to Zn and then increases again for higher atomic weights. In the present experiments we are dealing with the number of the β particles and not their ionization. The counting chambers were of thin material but either of different thickness of the same material or of different materials. The source of radiation was radium. Because of very marked scattering of γ rays and production of secondary radiation in air and objects about the room, this scattered and secondary radiation was also measured by interposing in the direct path of the γ rays a large amount of lead at places found by experiment to give the optimum effect of this radiation.[1] The count due to both, less the latter, gives the effect of the direct radiation. For the increasing small thickness the count increased to a maximum and then for greater thickness decreased again with the law of absorption for γ rays.

With materials of different atomic weights the count decreased from C but began to increase with Cu and increased for higher atomic weights. The minimum in the curve comes at somewhat smaller atomic weight than that reported by Hackett using the ionization method. In general the rising portion of the curve was not as steep. This may be expected when the number of β particles is counted instead of measuring their ionization since the velocity of the emerging β particles is affected by the material through which they pass and ionization increases with the decrease of the velocity.

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[1] Similar to Lawson's method, Akad. Wiss. Wien, 125, 1, 1916.

The High-Frequency Limits of X-Ray Spectra at Different Angles from the Cathode Stream.

By David L. Webster.

In a previous paper[1] evidence was collected from various sources on the distribution of x-rays relative to the cathode stream. It appeared that there were phenomena related to the Doppler effect, in such a way as to indicate: (1) that the general radiation, or continuous spectrum, is emitted by electrons most of which are moving at high speeds while emitting; (2) that the frequency of emission of each electron is determined by the energy it emits, apparently without regard to its surroundings, even though it may move through many atoms during the process of emission. This means that the electron must carry its whole emitting mechanism in itself, as a part of its own structure.

The data were not very definite, and the most doubtful point in the reasoning was a deduction from the experiments, to the effect that the high-frequency limit of the spectrum must be the same in all directions from the tube. This has since been confirmed for voltages up to 10 kilovolts by Wagner,[2] but it is denied for 80 kilovolts by Zecher,[3] who finds a difference of 11 per cent. in the high-frequency limits at angles of 60 and 150 degrees from the cathode rays. I have nearly completed apparatus for the study of certain phases of the distribution problem, and am able now to report preliminary results on this point, for tungsten, but unfortunately only up to 67 kilovolts because of some gas that has appeared in the tube. At this voltage, as well as at lower ones that have been tested, down to 38 kv., *no difference between the high-frequency limits has been found*. The accuracy of measurements to date is about one per cent., and the angles used are 55 and 145 degrees, nearly the same as Zecher's.

I think the explanation of Zecher's result lies partly in the fact that he used a photographic spectrum, in which the limit is always slightly indefinite, and even more in the fact that he used an induction coil to drive his tube. The present work was done by ionization, with a D.C. outfit of the kenotron type whose voltage fluctuation was a very small fraction of one per cent.

I hope soon to improve the accuracy of the measurements and extend their range.

Stanford University,
California.

A Generalization of the Electrodynamic Equations with Applications to the Structure of the Electron and to Radiationless Orbits.

By Leigh Page.

A year ago the author published[4] a generalization of the electrodynamic equations which took into account the possibility of rotation of electric fields.

[1] Phys. Rev., 13, 303, 1919.
[2] P. Z., 21, 621, 1920.
[3] Ann. der Phys., 63, 28, 1920.
[4] Proc. Nat. Acad. Sci., 6, p. 115, 1920.

A further generalization considers the possibility both of rotating electric and rotating magnetic fields. Each element of charge is supposed to be the origin of equal electric and magnetic fields. If the electron is of the Lorentz type, sufficient rotation of the electric fields of each element of charge about the normal to the surface annuls the intrinsic magnetic field outside the electron's surface, and the magnetic field inside the surface is responsible for an inward stress which just counterbalances the outward stress due to electrostatic repulsion, and accounts for the *stable* equilibrium of the electron.

If the fields of the elements of charge on the surface of an accelerated electron have a small additional rotation parallel to the acceleration, the radiation field vanishes, and radiationless orbits are possible. Details are contained in a more extended paper which has been submitted to the PHYSICAL REVIEW for publication.

YALE UNIVERSITY.

THE RESISTANCE AND INDUCTANCE OF A THREE-PHASE CABLE.

BY HARVEY L. CURTIS.

AN integration method of computing alternating current resistance and inductance was presented before the Physical Society[1] at its meeting in April, 1920. This method has now been applied to the case of a three-phase cable of cylindrical wires. The resistance and inductance of one of the wires have been computed when balanced three-phase currents are flowing in the cable. The results show that the resistance is more and the inductance less than in a similar single-phase cable.

In the case of a single-phase cable, the change of resistance with frequency is approximately given by the formula

$$\frac{R}{R_0} = \frac{1 + \frac{\lambda^2}{6} + \frac{\lambda^4}{480} + \cdots}{1 + \frac{\lambda^2}{12} + \frac{\lambda^4}{1440} + \cdots} + \frac{\lambda^2 a^2}{s^2} \frac{1 + \frac{\lambda^2}{24} + \frac{\lambda^4}{8640} + \cdots}{1 + \frac{\lambda^2}{2} + \frac{\lambda^4}{96} + \cdots}$$

$$+ \text{terms in } \frac{a^4}{s^4}, \frac{a^6}{s^6}, \text{ etc.}$$

Where R_0 = the direct current resistance.

R = the alternating current resistance at the frequency f.

$\lambda = \frac{2\pi^2 f a^2}{\sigma}$

σ = the resistivity of the material of which the cable is made.

a = the radius of one of the wires.

s = the distance between the centers of the two wires of the cable.

The first term of the second member is the change of resistance in one wire when the return is at infinity. The second term gives the effect of the adjacent wire and may be called the proximity term since it depends upon the distance

[1] PHYS. REV., 2d Series, XV., p. 528. A more complete discussion is given in Scientific Paper No. 374 of the Bureau of Standards.

between the wires. The terms a^4/s^4, a^6/s^6, etc., never affect the result by as much as 10 per cent, and hence can be neglected for approximate calculations.

In a three-phase cable, the formula for change of resistance is approximately

$$\frac{R}{R_0} = \frac{1 + \frac{\lambda^2}{6} + \frac{\lambda^4}{480} \cdots}{1 + \frac{\lambda^2}{12} + \frac{\lambda^4}{1440} \cdots} + \frac{3\lambda^2 a^2}{2s^2} \frac{1 + \frac{\lambda^2}{24} + \frac{\lambda^4}{8640} \cdots}{1 + \frac{\lambda^2}{2} + \frac{\lambda^4}{96} \cdots}$$

$$+ \text{terms in } \frac{a^4}{s^4}, \frac{a^6}{s^6}, \text{ etc.}$$

The symbols have the same meaning as in the preceding case, except that s is now the length of a side of the equilateral traingle which is formed by joining the centers of the three wires of the cable. The first term of the right hand member is the same as in the single-phase cable, but the proximity term is 50 per cent. greater.

The inductance is also very similar for the two cases. For the single-phase cable the approximate formula is

$$L = 2l \log \frac{s}{a} + \frac{l}{2} \frac{1 + \frac{\lambda^2}{24} + \frac{\lambda^4}{4320} + \cdots}{1 + \frac{\lambda^2}{12} + \frac{\lambda^4}{1440} + \cdots}$$

$$- \frac{2l\lambda^2 a^2}{3s^2} \frac{1 + \frac{\lambda^2}{40} + \frac{\lambda^4}{10080} + \cdots}{1 + \frac{\lambda^2}{2} + \frac{\lambda^4}{96} + \cdots}$$

$$- \text{terms in } \frac{a^4}{s^4}, \frac{a^6}{s^6}, \text{ etc.}$$

where L is the inductance of one wire of the cable, and l is the length of the cable. For the three-phase cable the last or proximity term is 50 per cent. greater than in the above formula.

The above formulas show that the proximity effect is approximately 50 per cent. greater in the three-phase cable than in the single-phase cable. However, this is not exact, since the coefficients of the terms in a^4/s^4, a^6/s^6, etc., have a different numerical ratio than the term in a^2/s^2. For exact computations, the complete formulas must be consulted.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
April 6, 1921.

THICKNESS OF STRATIFIED SOAP FILMS.

BY P. V. WELLS.

NEWTON noticed two black spots in the soap film, each layer of uniform thickness with sharp boundaries. Kelvin showed that such stratification indicated the range of molecular forces. Johonnot noticed at least five different

strata, but Perrin, using the microscope with reflected light, as in metallography, observed hundreds of such "plages" increasing in thickness by uniform steps, each plage a multiple of the unit plage in thickness.

Measuring the thickness photometrically and colorimetrically, Perrin's multiple "plage" law is verified, the unit plage being 4×10^{-7} cm. thick, the same as the black spot. The results, considered in relation to the recent work on the spreading of oil films on water, suggest that the elementary layer is a bimolecular film of oleic acid, the molecules being bound together by their active groups, their inactive tails forming the external surfaces of the film. Such a double layer of molecules is of the proper thickness, is stable, and would show many of the phenomena of stratified films. For instance discs of unit thickness are sometimes observed floating about upon a plage with brownian motion in equilibrium, instead of pulling together into a globule as would be expected. A detailed study of the life history of stratified films adduce much evidence supporting this structure.

If the black spot (plage 1) is bimolecular, and plages of greater thickness (plages 2, 3, 4, $\cdots$ n, etc.) consist of superposed strata each two molecules thick, such a theory may be of interest and importance in the problem of lubrication at low speeds and high pressures. Besides the monomolecular film of oleic acid firmly attached to the metal surfaces, there may be strata of bimolecular thickness, forming inactive lamina between the bearing surface, and behaving quite differently from the lubricant in bulk. Recent progress in lubrication renders such concepts plausible.

BUREAU OF STANDARDS.

THE TEMPERATURE OF AN ELECTRICALLY HEATED FILAMENT.

BY CHESTER SNOW.

THE temperature distribution of a fine wire, electrically heated by a current passing along it, and surrounded by a poorly conducting medium, has been studied for the temperature interval 0 to about 600° C. The temperature-coefficient of resistance and the thermal conductivity of the wire may be considered constant over this range of temperatures. The wires were connected to massive terminals which were practically unheated by the current.

It was found by experiments on relatively long wires surrounded by air at various pressures, and by finely powdered material such as flour, that the effect of the surrounding medium upon the wire temperature could be accounted for by assuming the heat flow from wire to medium proportional to the difference in temperature between the wire and the nearest layer of the medium. Such a temperature difference existed, at least in a statistical sense.

Using this fact and the known conductivity of the wire, expressions have been obtained mathematically for the wire temperature at any point. The following cases were considered:

1. Thermal equilibrium when the filament formed part of a circuit in which the current was maintained constant.

2. Thermal equilibrium when the filament formed part of a circuit in which the electromotive force was maintained constant.

3. Constant current. Time rate of heating of the wire and the medium.

In case 1, formulas have been derived by which the lateral heat flow to the medium, and the temperature of the wire and medium, may be computed from observations of wire resistance as a function of the current. The variations of lateral heat loss with diameter of wire were found by experiment to be in accordance with the theory. Convenient formulas have been obtained to correct for end conduction in long or short wires.

In case 2, the current obtained depended upon the temperature of the filament. Graphical methods were devised by which temperature of the middle of the wire was plotted as function of the dimensions or of the constants of the material of the wire. The dimensions for maximum wire temperature with a given electromotive force were derived. The effect of varying the material of the wires, upon the equilibrium temperatures, was also considered. The effect of varying the thermal contact between the wire and medium, upon the equilibrium temperature of each was also studied. This factor was found to be just as important as the electrical and thermal constants of the wire, in determining the final wire temperature produced by a given electromotive force. The sensitiveness of the resistance variation with current in high vacua suggests the possibility of using this device as an electric vacuum gauge.

In case 3, the time rates of rise of wire and medium were considered. It was assumed that the above law of interchange of heat between wire and medium held even when the temperatures were varying. The heat flow in the medium was considered to be practically radial. The equations of heat conduction for the two mutually influencing thermal bodies (wire and medium) led to an integral equation, the kernel of which was obtained as a Fourier-Bessel expansion. It was found that a single term, of exponential type, could be chosen which would represent this kernel with sufficient approximation and which led to a simple solution. By this means it was found that the wire temperature $U(t)$ at any point, at time t, and the corresponding temperature of the adjacent medium $V(t)$ were given by

$$U(t) = \frac{U(\infty)}{b(\alpha_1 - \alpha_2)}\left[\alpha_2(\alpha_1 - b)(1 - e^{-\alpha_1 t}) + \alpha_1(b - \alpha_2)(1 - e^{-\alpha_2 t})\right]$$

and

$$V(t) = \frac{V(\infty)}{\alpha_1 - \alpha_2}\left[\alpha_1(1 - e^{-\alpha_2 t}) - \alpha_2(1 - e^{-\alpha_1 t})\right],$$

where t was the time measured from the instant of closing the circuit and where $U(\alpha)$ and $V(\alpha)$ were the final temperatures of wire and medium, and α_1, α_2, and b were given functions of the thermal and electrical constants of the wire and of the thermal contact of the wire and its surrounding medium. All but two of these constants can be obtained from steady-state measurements. The remaining two can be obtained from oscillograph measurements, of current

and voltage, from which the temperature-time curve of the wire may be computed. The formulas indicate that the wire temperature rises with an appreciable initial slope to a limiting value, $U(\infty)$, without passing through any maximum value. The medium rises more slowly with an initial slope of zero, to a final temperature lower shan the wire.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
April 18, 1921.

STANDARD WAVE-LENGTHS AND CONSTANT FREQUENCY DIFFERENCES IN THE SPECTRA OF INERT GASES.

BY W. F. MEGGERS.

THE spectra of the inert gases (helium, neon, argon, krypton and xenon) are known to consist of sharp lines which represent wave-lengths of a very high degree of homogeneity.[1] In fact, the spectral lines obtained from the heavier gases are the narrowest regions of spectral emission known at present. This makes them susceptible of extreme precision in wave-length comparisons by means of interferometers. Precision, reproducibility and convenience are the properties of these lines which recommend them for use as standards of wave-length, or ultimately as fundamental standards of length. The Bureau of Standards has, from time to time, carefully compared wave-lengths in the spectra of the inert gases with the wave-length of the red radiation from cadmium which is the international primary standard of wave-length. The results from 21 lines in the He spectrum (2945–7281A) and 55 lines in the spectrum of neon (3369–8495A) have already been published.[2] Work has recently been completed on 51 lines (3948 to 8521A) in the argon spectrum, 18 lines in krypton (4723–7601A), and 12 lines in the spectrum of xenon (4500–4923A). All these wave-length determinations were made by the interferometer method devised by Fabry and Perot,[3] which consists in accurately obtaining, from circular interference fringes, the number of standard and approximately known wave-lengths contained in a fixed distance between two parallel interferometer plates. The probable error in the final wave-lengths is about one or two thousandths of an angstrom unit. In other words, the wave-lengths are correct to one part in several millions.

Frequency differences (*i.e.*, differences in the number of waves per cm. vacuum) are very interesting in the light of these accurate measurements. In the case of neon it was shown that 37 of the standard lines were associated in quadruplets having frequency differences[4] which were constant within the

[1] Fabry and Buisson, J. de Phys., (5), 2, p. 442, 1912.

[2] B. S. Bulletin 14, p. 159, 1917, and 14, p. 765, 1918.

[3] Fabry and Perot, Ann. d. Chim., 25, p. 110, 1902.

[4] These regularities in the neon spectrum have recently been arranged in spectral series by Meissner (Ann. d. Phys., 58, p. 333, 1919), and more completely by Paschen (Ann. d. Phys., 60, p. 405, 1919).

accuracy of the wave-length measurements, and leading to the following frequency relations between members of quadruplets:

$$\nu_2 = \nu_1 + 1070.077,$$
$$\nu_3 = \nu_2 + 359.956,$$
$$\nu_4 = \nu_3 + 417.449.$$

Similar quadruplets[1] in which 28 of the standard argon wave-lengths find places show that the frequency differences are again constant within 1 part in several millions and the following frequency relations are derived:

$$\nu_2 = \nu_1 + 846.162,$$
$$\nu_3 = \nu_2 + 803.075,$$
$$\nu_4 = \nu_3 + 606.837.$$

Eight of the standard krypton lines are found to constitute pairs whose mean frequency difference is 945.028, and no value deviates from the mean by more than one part in 9 millions.

Fainter lines in the xenon spectrum must be measured with the interferometer before the constancy of frequency differences[2] in its spectrum can be tested.

Bureau of Standards,
April 18, 1921.

Note on the Diurnal Variation of the Atmospheric-Electric Potential-Gradient.

By S. J. Mauchly.

Since the summer of 1915 observations have been made aboard the *Carnegie* to determine the nature of the diurnal variation of the atmospheric-electric elements over the oceans. For the potential-gradient the general procedure is to make a set of 20 observations during each of 24 consecutive hours. The observations for such a set require about 20 minutes and their mean value is referred to the mean time for the set. From reductions based on the observations made prior to April, 1916, it appeared that the diurnal variation of the potential-gradient over the oceans probably did not differ much from that which has been found at many land stations; that is, they indicated two rather pronounced maxima and two minima during a 24-hour period. However, very few data were available from oceans other than the Pacific, and a large percentage was derived from series of observations which were terminated by the advent of unfavorable weather.

The largely increased amount of material which has accumulated now makes it possible to reject all data corresponding to less than a 24-hour run and still have 45 complete 24-hour runs available. The data for each run, therefore, corresponds to an actually occurring sequence of phenomena, and

[1] Rydberg, Astroph. J., 6, p. 345, 1897. Paulson, Physik. Z., 15, p. 831, 1914. Meissner, Physik. Z., 17, p. 551, 1916.

[2] Paulson, Astroph. J., 40, p. 298, 1914.

the mean results are free from the errors which would result from combining the results of partial series of observations.

Of the 45 diurnal-variation runs referred to, 30 were made in the Pacific, 5 in the Atlantic, and 10 in the Indian Ocean. The means corresponding to the separate oceans, as derived from 39 runs, are represented in Fig. 1. They show: (1) That the mean diurnal-variation curves for the Pacific, Atlantic and Indian oceans are similar in form; (2) that the principal component of the variation consists of a 24-hour "wave"; and (3) that the phase of this wave differs for the several oceans by amounts which correspond approximately to the differences between the mean longitudes of the points at which the observations were made.

Since the curves of Fig. 1 suggest the simultaneous occurrence of maximum phase over all three oceans, it was decided to refer the results of each run to Greenwich Mean Time (civil), and re-compute the means for the separate oceans on this basis. The results are shown in Fig. 2, together with a curve which includes the data from 6 new runs. The differences between the several curves of Fig. 2 are of course not to be thought of as representative of separate characteristics, since the smoothness of the respective curves is seen to be directly related to the number of component runs.

The curves of Fig. 2 show a decided similarity to land results for high latitudes and also to many of the winter curves obtained in ordinary latitudes. If differences in local mean time are taken into account it appears that, in addition to their similarity of form, many of these land curves are *simultaneously in the same phase as the general ocean curve* of Fig. 2. For the summer, however, as is well known, most land stations show, in addition to the above 24-hour wave, a decided secondary wave which seems to occur in general at about the same local mean time at different stations.

The results of the 45 complete runs representing a general distribution over the accessible oceans indicate, therefore, as a preliminary result, that *the chief component of the diurnal variation of the potential-gradient over the major portion of the Earth (especially the oceans) is a "wave" of 24-hour period which occurs simultaneously in all localities.* While the preliminary investigations show that for the oceans this general statement holds for all latitudes and seasons, there are evidences of secondary variations which are being investigated, and whose nature will become more certain as the results of additional observations become available.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
WASHINGTON, D. C., April, 1921.

PECULIAR ICE FORMATIONS.

BY HERBERT GROVE DORSEY.

THE first of these formations of ice occurred on the night of December 15, 1916. The night was clear and calm and some water had been left in a pan placed on a chair on a small uncovered porch. The temperature the following morning was 22° Fahrenheit.

The pan was an ordinary tin coated iron pan, 17 inches in diameter at the top, 12 inches at the bottom and six inches deep. About one inch of unboiled hydrant water was in the pan, and when it froze a thin irregular column of ice was formed near the side of the pan next to the back of the chair.

Fig. 1 shows a front view of the formation, Fig. 2 a side view of it and Fig. 3 is a larger photograph to show the details. The photographic dark slide was placed back of it to give contrast. The steel scale was in the plane of the edge of the ice column and melted the ice so that it was resting on the bottom. It appears nearer than the column because the ice is melted in front of the scale.

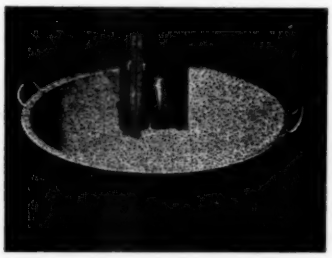

Fig. 1.

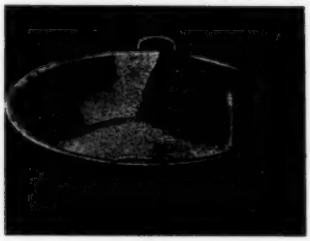

Fig. 2.

Fig. 3.

Fig. 4.

The maximum height of the ice column was four and one half inches and the breadth at the bottom was three inches, narrowing irregularly to one and three eighths inches near the middle and to one inch at the top. The average thickness was about one half inch, and instead of being vertical it leaned towards the center of the pan. It will be noticed that while the air bubbles in the pan were in vertical lines, they arose near the middle of the column and then separated out laterally into almost horizontal lines. Near the top the air bubbles were grouped together so closely that it gave the appearance of white ice or snow; the rest of the ice in the pan and in the formation was transparent except for the bubbles.

Water had been frozen in this same pan several nights previously and after the unusual formation occurred, it was tried under nearly similar conditions many times that winter and the following winter, but the only approach to

anything so unusual occurred four nights later in a small wooden chopping bowl about ten inches in diameter. As shown in Fig. 4 the formation in this case was almost egg-shaped, the maximum length of the egg being seven eighths of an inch. The bowl was cracked when this occurred.

GLOUCESTER, MASS.,
April 1, 1921.

ON THE EXPLODED-WIRE SPECTRUM OF CALCIUM.

BY R. A. SAWYER AND A. L. BECKER.

THE spectra of exploded wires of calcium and several other elements have been photographed.

By expending constant power upon wires of varying mass, or varying power upon wires of constant mass, a series of explosions of varying temperature has been obtained. It has been possible to obtain spectra in which the relative intensities of the three Ca lines, H, K and λ 4227 vary as they vary in the spectra of the different classes of stars.

These three Ca lines also appear more prominently in the spectra of exploded wires of other metals than the spectra of the metals themselves. The amount of calcium present must be very minute. It is therefore probable that the calcium lines particularly H and K are tremendously enhanced by the very high temperature of the explosion. The intense Ca lines of the solar flash spectra have seemingly required considerable quantities of the heavier Ca to be above the hydrogen layers of the solar atmosphere. The present results explain this anomaly as primarily a temperature effect.

UNIVERSITY OF MICHIGAN.

ON THE RADIATION OF ENERGY FROM COILS.

BY ARTHUR GORDON WEBSTER.

INSTEAD of the usual approximations, exact formulæ are developed for the radiation from a coil in which the density of current is constant, both for rectangular and for circular coils. This paper will be published in the Proceedings of the National Academy of Sciences.

CLARK UNIVERSITY.

ON THE VIBRATIONS OF GUN-BARRELS.

BY ARTHUR GORDON WEBSTER.

THE theory of the propagation of the couple that is developed by the kick of the gun is considered, and an estimate of the time taken to arrive at the end of the barrel is obtained. The method of finding the error of sighting due to the elastic wave is indicated. This paper will be published in the Proceedings of the National Academy of Sciences.

CLARK UNIVERSITY.